

Honors Geometry Placement Examination

Comprehensive Final-Exam-Level Assessment

Name: _____

Date: _____

Time allowed: 2 hours 30 minutes • **Total:** 150 points, plus 10 extra-credit points

Instructions. Show all work; answers without supporting work receive no credit. Diagrams are not drawn to scale. Exact answers (radicals, π) are required unless the problem says "round." A scientific calculator is permitted; no notes or formula sheets. For proofs, you may use two-column, paragraph, or flow format unless a format is specified. Justify each statement with a definition, postulate, or theorem.

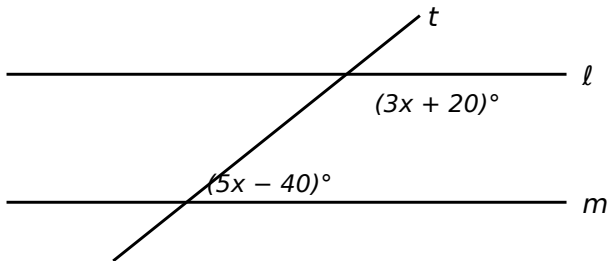
Part I — Logic, Lines, and Angles (14 points)

Foundations of geometric reasoning.

1. (6 pts) Consider the conditional statement: "If a quadrilateral is a rhombus, then its diagonals are perpendicular."

- (a) Write the converse, the inverse, and the contrapositive.
- (b) Determine the truth value of each of the four statements. For each false statement, give a specific counterexample.

2. (4 pts) In the figure, $\ell \parallel m$ and t is a transversal. The two marked angles measure $(3x + 20)^\circ$ and $(5x - 40)^\circ$, positioned as shown (same-side interior angles). Find x and the measure of each marked angle. Name the theorem that justifies your equation.



3. (4 pts) An exterior angle of a triangle measures $(7x - 4)^\circ$, and its two remote interior angles measure $(3x + 10)^\circ$ and $(2x + 30)^\circ$. Find the measures of all three interior angles of the triangle.

Part II — Proofs (40 points)

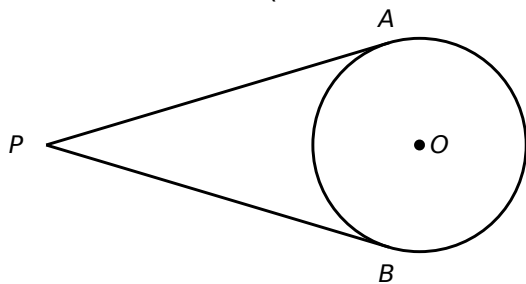
Write complete proofs. Every statement must have a reason.

4. (8 pts) **Given:** Quadrilateral ABCD with $AB \parallel DC$ and $AB \cong DC$; diagonal AC is drawn.
Prove: $\triangle ABC \cong \triangle CDA$, and hence $AD \parallel BC$ (i.e., ABCD is a parallelogram).

5. (8 pts) **Given:** $\triangle ABC$ with $AB \cong AC$; ray AD bisects $\angle BAC$ and intersects BC at D.
Prove: $AD \perp BC$ and D is the midpoint of BC.

6. (8 pts) **Coordinate proof.** Quadrilateral ABCD has vertices A(0, 0), B(5, 0), C(8, 4), and D(3, 4). Prove that ABCD is a rhombus but **not** a square. Then verify that its diagonals are perpendicular.

7. (8 pts) **Given:** PA and PB are tangent to circle O at points A and B, respectively.
Prove: $PA \cong PB$. (Hint: draw auxiliary segments.)



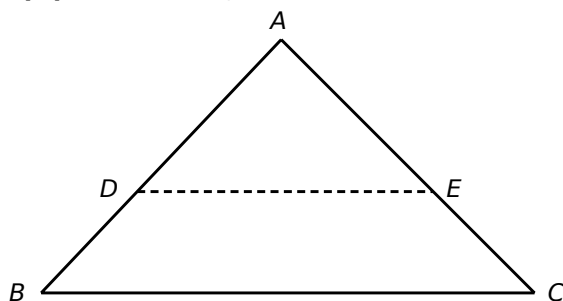
8. (8 pts) **Indirect proof.** Prove that a triangle can have at most one obtuse angle. State your assumption clearly and show that it leads to a contradiction.

Part III — Similarity and Right Triangles (19 points)

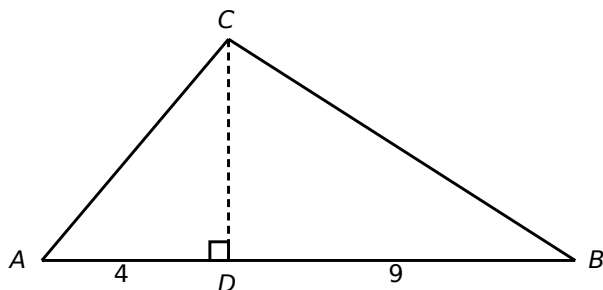
9. (4 pts) In $\triangle ABC$, points D and E lie on AB and AC with $DE \parallel BC$. $AD = 6$, $DB = 4$, and $AE = 9$.

(a) Find EC.

(b) If $BC = 15$, find DE.



10. (5 pts) In right triangle ABC, $\angle ACB = 90^\circ$ and CD is the altitude to hypotenuse AB, with $AD = 4$ and $DB = 9$. Find CD, AC, and BC. Express all answers in simplest radical form.



11. (4 pts) Find the exact value of x in each case:

(a) A 45° - 45° - 90° triangle has hypotenuse 10; x is the length of a leg.

(b) A 30° - 60° - 90° triangle has longer leg 12; x is the length of the hypotenuse.

12. (6 pts) From point A on level ground, the angle of elevation to the top of a radio tower is 32° . After walking 50 meters directly toward the tower to point B, the angle of elevation is 47° . Find the height of the tower to the nearest tenth of a meter. Include a labeled diagram in your solution.

Part IV — Trigonometry (32 points)

13. (8 pts) The ambiguous case. In $\triangle ABC$, $m\angle A = 35^\circ$, $a = 10$, and $b = 16$.

- (a) Show that two distinct triangles satisfy these conditions.
- (b) For *each* triangle, find $m\angle B$, $m\angle C$, and c . Round angles to the nearest tenth of a degree and sides to the nearest hundredth.

14. (8 pts) In $\triangle PQR$, $PQ = 7$, $PR = 9$, and $m\angle P = 60^\circ$.

- (a) Use the Law of Cosines to find QR in simplest radical form.
- (b) Find the area of $\triangle PQR$ using the formula $\text{Area} = \frac{1}{2} \cdot ab \cdot \sin C$.
- (c) Verify your area by applying Heron's formula to the three side lengths. Show that the two results agree.

15. (8 pts) Prove each trigonometric identity. Work with one side only and justify each step.

(a) $\sin \theta / (1 - \cos \theta) + (1 - \cos \theta) / \sin \theta = 2 \csc \theta$

(b) $\sec \theta - \cos \theta = \sin \theta \cdot \tan \theta$

16. (4 pts) Evaluate exactly, without a calculator:

$$\sin 60^\circ \cdot \cos 30^\circ - \sin 30^\circ \cdot \cos 60^\circ + \tan^2 45^\circ - \sec 60^\circ$$

17. (4 pts) Two sides of a triangle measure 8 cm and 12 cm, and the included angle measures 115° . Find the area of the triangle to the nearest tenth of a square centimeter.

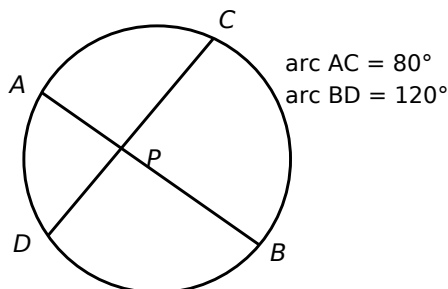
Part V — Circles (22 points)

18. (6 pts) Chords AB and CD of a circle intersect at interior point P. Arc AC = 80° and arc BD = 120° .

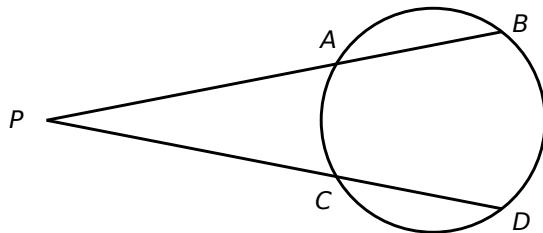
(a) Find $m\angle APC$.

(b) An inscribed angle intercepts arc BD. Find its measure.

(c) If instead two secants met at a point *outside* the circle and intercepted arcs of 120° and 80° , what would be the measure of the angle formed at the external point?



- 19.** (6 pts) From external point P, secant PAB and secant PCD are drawn to a circle (A between P and B; C between P and D). $PA = 4$, $AB = 8$, and $PC = 6$.
- (a) Find CD.
- (b) A tangent from P touches the circle at T. Find PT in simplest radical form.



- 20.** (6 pts) The endpoints of a diameter of circle C are $(-2, 3)$ and $(6, -3)$.
- (a) Write the equation of the circle in standard form.
- (b) Verify that the point $(5, 4)$ lies on the circle.
- (c) Write the equation of the line tangent to the circle at $(5, 4)$.
- 21.** (4 pts) A circle has radius 9 cm. A central angle measures 140° . Find the exact arc length and the exact area of the sector determined by this angle (answers in terms of π).

Part VI — Polygons, Area, and Solid Geometry (23 points)

- 22.** (5 pts) Answer both parts:
- (a) Each interior angle of a regular polygon measures 156° . How many sides does the polygon have?
- (b) Find the sum of the interior angle measures of a convex 11-gon.

- 23.** (5 pts) A triangle has sides 13, 14, and 15.
- (a) Use Heron's formula to find its area.
 - (b) Use your answer to find the length of the altitude drawn to the side of length 14.
- 24.** (4 pts) A regular hexagon has side length 12. Find its exact apothem and its exact area.
- 25.** (4 pts) Two similar pyramids have corresponding edges in the ratio 2 : 5. The smaller pyramid has volume 64 cm^3 . Find the volume of the larger pyramid, and state the ratio of their surface areas.
- 26.** (5 pts) A solid metal sphere of radius 6 cm is melted down and recast into a solid cone whose base radius is also 6 cm (no metal is lost). Find the height of the cone.

Part VII — Challenge Problems (Extra Credit, 10 points)

Attempt these only after completing Parts I–VI.

- 27.** (5 pts) Cyclic quadrilateral ABCD is inscribed in a circle with $AB = 5$, $BC = 7$, $CD = 9$, and $DA = 3$. Recall that opposite angles of a cyclic quadrilateral are supplementary.
- (a) By applying the Law of Cosines to diagonal AC in triangles ABC and ACD, find $\cos B$.
 - (b) Find the exact length of AC.
 - (c) Find the exact area of ABCD.

- 28.** (5 pts) Prove that the median drawn to the hypotenuse of a right triangle is half as long as the hypotenuse. (A coordinate proof is recommended: place the right angle at the origin with legs along the axes, using coordinates such as $(0, 2a)$ and $(2b, 0)$.)

End of examination. Check that you have shown supporting work for every problem.