

Work Problems: Think in ONE Hour

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How much of the job gets done in a single hour?

The big idea

A work problem tells you how long a whole job takes. The trick is to flip that around and ask: **how much of the job gets done in ONE hour?** That's the worker's **rate** - just like miles per hour, but it's **jobs per hour**.

If a dragon can clean the whole cave in **4 hours**, then in 1 hour it cleans **$\frac{1}{4}$ of the cave**.

Picture it: the job is a bar

The whole cave = the whole bar. The dragon takes 4 hours, so cut the bar into 4 equal pieces - one piece per hour.



After 1 hour: 1 piece done = $\frac{1}{4}$ of the job. **After 3 hours:** 3 pieces = $\frac{3}{4}$. **After 4 hours:** $\frac{4}{4}$ = the whole job. Done!

Takes T hours for the whole job -> does $\frac{1}{T}$ of the job each hour

Try a few

A troll paints a fence in 6 hours.

$\frac{1}{6}$ of the fence per hour

A goblin digs a tunnel in 10 days.

$\frac{1}{10}$ of the tunnel per day

A witch brews a cauldron in 3 hours.

$\frac{1}{3}$ of the cauldron per hour

A unicorn eats a hay bale in 2 days.

$\frac{1}{2}$ of the bale per day

TIP Whole job in T hours means $\frac{1}{T}$ per hour. The number of hours goes on the **BOTTOM** of the fraction.

Your Turn: One Hour

Write each worker's share of the job per hour

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Name: _____

Part A: Fill in the rate

1. A dragon fills the moat in 5 hours. Per hour: _____
2. A monster mows the lawn in 8 hours. Per hour: _____
3. A wizard copies a spell book in 12 days. Per day: _____
4. Two gnomes build a wall in 9 hours (working together). Per hour: _____
5. A giant eats a whole pumpkin patch in 4 hours. How much in 3 hours? _____
6. A troll digs $\frac{1}{7}$ of a ditch every hour. How many hours for the whole ditch? _____

Part B: Shade the bar

A goblin takes **6 hours** to sort a pile of gems. Cut the bar into the right number of pieces (draw the lines), then shade how much is done after **4 hours**.



Fraction done after 4 hours: _____ Fraction still left: _____

If the goblin has a helper who does $\frac{1}{6}$ per hour too, how much do they do TOGETHER in 1 hour? _____

TIP Part B last question is the key to the next page: two workers together just ADD their shares.

Working Together

Add the shares, then flip

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The three steps

- 1 Find each worker's share of the job **per hour** ($1 / \text{their time}$).
- 2 **Add** the shares. That's how much they do together in one hour.
- 3 **Flip** that fraction to get the hours for the whole job.

Example: dragon cleans the cave in 6 h, griffin in 3 h. Together?

Step 1: Dragon does $1/6$ per hour. Griffin does $1/3$ per hour.



dragon: 1 piece per hour



griffin: 2 pieces per hour ($1/3 = 2/6$)

Step 2: Together in one hour: $1/6 + 1/3 = 1/6 + 2/6 = 3/6 = 1/2$ of the cave. (The griffin's $1/3$ is 2 pieces of a 6-piece bar - same-size pieces so you can add.)



together: 3 pieces per hour = $1/2$

Step 3: Half the job per hour means the whole job takes **2 hours**. (Flip $1/2$ to get $2/1$.)

$$1/6 + 1/3 = 1/2 \text{ per hour} \rightarrow 2 \text{ hours together}$$

When the flip isn't a whole number

A troll paints a fence in 4 hours, a goblin in 6 hours. $1/4 + 1/6 = 3/12 + 2/12 = 5/12$ per hour. Flip: **$12/5$ hours = 2 and $2/5$ hours** = 2 hours 24 minutes. *Sanity check: together they must beat the faster worker's 4 hours - and 2.4 does.*

TIP Together is always FASTER than the fastest worker alone. If your answer is bigger than the smallest time, something flipped the wrong way.

Your Turn: Working Together

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Shares per hour -> add -> flip

Name: _____

- 1 A dragon clears a field in 2 hours. A giant clears it in 2 hours. How long together?

Easy one to start: what's $1/2 + 1/2$?

Together: _____

- 2 A witch brews a potion in 3 hours. Her apprentice takes 6 hours. How long together?

$1/3 + 1/6 = ?$

Together: _____

- 3 A troll builds a wall in 4 hours. A gnome takes 12 hours. How long together?

Find a common denominator.

Together: _____

- 4 One monster eats a pie in 10 minutes. Another eats the same pie in 15 minutes. How long for both to eat one pie together?

$1/10 + 1/15$ - common denominator is 30.

TIP Check every answer: it must be less than the fastest worker's time alone.

Work Problem Twists

Same idea, more interesting questions

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Twist 1: One worker leaves (or starts late)

A dragon (6 hours alone) and a griffin (3 hours alone) start cleaning together. After **1 hour** the griffin leaves. How long does the dragon take to finish?

Together for 1 hour: $1/6 + 1/3 = 1/2$ done. **Left over:** $1 - 1/2 = 1/2$ of the cave.

Dragon alone: does $1/6$ per hour, needs to do $1/2 = 3/6$. That's **3 more hours**. Total time: $1 + 3 = 4$ hours.

Twist 2: Find the missing worker

A troll and a goblin paint a fence together in **2 hours**. The troll alone takes **3 hours**. How long would the goblin take alone?

Together per hour: $1/2$. **Troll per hour:** $1/3$. **So goblin per hour** $= 1/2 - 1/3 = 3/6 - 2/6 = 1/6$.

Flip: the goblin alone takes **6 hours**. *Subtract shares to find the one you don't know.*

Twist 3: Filling and draining (one works AGAINST the other)

A pipe fills a moat in **4 hours**. A leak drains a full moat in **12 hours**. With both, how long to fill the moat?

Pipe: $+1/4$ per hour. **Leak:** $-1/12$ per hour. **Net:** $1/4 - 1/12 = 3/12 - 1/12 = 2/12 = 1/6$ per hour.

Flip: **6 hours**. The leak slows it down from 4 to 6 - makes sense!

Twist 4: More than one job

If together they do $1/2$ of a job per hour, then **3 jobs** takes $3 \times 2 = 6$ hours. Find the time for ONE job, then multiply.

TIP Every twist is still 'shares per hour'. Helpers ADD, leaks SUBTRACT, and whatever is left over gets finished at the remaining rate.

Your Turn: Twists

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Shares per hour, then think carefully about who is working when

Name: _____

- 1** A monster and a troll can dig a pit together in 4 hours. The monster alone takes 6 hours. How long would the troll take alone?

Subtract shares.

Answer: _____

- 2** A dragon fills a cauldron in 3 hours. A crack drains a full cauldron in 6 hours. With both happening, how long to fill it?

Fill minus drain.

Answer: _____

- 3** A witch (4 hours alone) and a wizard (12 hours alone) brew together for 2 hours. Then the witch leaves. How much longer does the wizard need to finish?

How much is done after 2 hours? How much is left?

Answer: _____

TIP Write a 'per hour' line for every worker before you do anything else.

Work Problems Cheat Sheet

Everything on one page

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ONE WORKER

Whole job in T hours $\rightarrow 1/T$ of the job per hour
In h hours, they do h/T of the job

Cave in 4 h $\rightarrow 1/4$ per hour

TWO WORKERS TOGETHER

Add the shares: $1/A + 1/B =$ share per hour
Flip to get the time for the whole job

$1/6 + 1/3 = 1/2 \rightarrow 2$ hours

MISSING WORKER

Together share - known share = missing share
Then flip

$1/2 - 1/3 = 1/6 \rightarrow 6$ hours

SOMEONE LEAVES

Work done so far = share x hours
Left over = $1 -$ done. Finish at the remaining rate.

FILL vs DRAIN

Fill is +, drain is -. Net share = fill - drain
Then flip

$1/4 - 1/12 = 1/6 \rightarrow 6$ hours

Sanity check: together is always faster than the fastest worker alone.