

What Is a Remainder?

The part that doesn't fit into equal groups

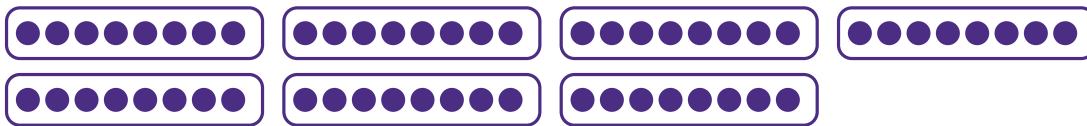
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The idea

When you divide and the groups don't come out even, the **remainder** is what's left over. It is always **smaller than the number you divided by** - if it weren't, you could fill another group!

Example: 58 cookies, boxes of 8

Fill boxes of 8 until you can't fill another one. 7 boxes use 56 cookies. The 2 that don't fit are the remainder.



left over: ●●

$$58 / 8 = 7 \text{ remainder } 2$$

7 boxes (the quotient), 2 cookies left over (the remainder). We also write it: $58 / 8 = 7 \text{ R}2$

The check: groups x size + leftover = total

$8 \times 7 + 2 = 56 + 2 = 58$. It works! You can always check a remainder this way.

If the remainder is **0**, the division came out even - the divisor is a **factor** of the number (that's exactly what we learned in the Factors guide).

TIP The remainder is never as big as the divisor. Dividing by 8? The remainder must be 0, 1, 2, 3, 4, 5, 6, or 7. If you get 8 or more, fill another box!

Meet the Modulo Operator

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'mod' is a short way to say 'the remainder when you divide by'

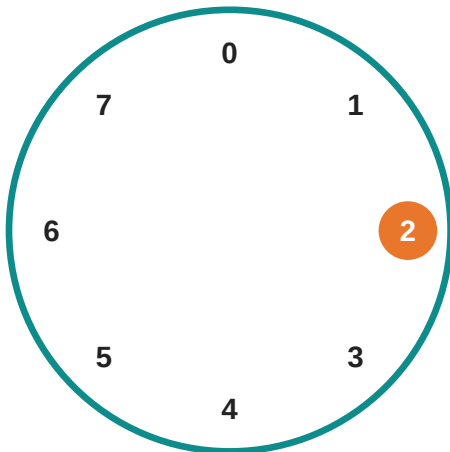
The definition

a mod n means **the remainder when a is divided by n**.

$58 \bmod 8 = 2$ (because $58 / 8 = 7 \text{ R}2$). $20 \bmod 5 = 0$ (because 5 goes into 20 evenly).

Say it: 'fifty-eight mod eight is two'. On a calculator or computer it's often written **58 % 8**.

Mod is a clock that wraps around



A 'mod 8' clock has only 0 to 7. Count 58 steps around from 0 and you land on **2** - every 8 steps brings you back to 0, so only the leftover 2 steps matter.

Try these

$$17 \bmod 5 = 2$$

$$17 = 5 \times 3 + 2$$

$$30 \bmod 6 = 0$$

$$30 = 6 \times 5 + 0$$

$$9 \bmod 10 = 9$$

$$9 \text{ is smaller than } 10$$

$$100 \bmod 7 = 2$$

$$100 = 7 \times 14 + 2$$

$$25 \bmod 4 = 1$$

$$25 = 4 \times 6 + 1$$

Notice: $9 \bmod 10$ is just 9. If the number is smaller than what you divide by, it IS the remainder.

Think about it: fill in your answers

1 What is **35 mod 8**? _____

2 If I add 8 to 35 (that's 43), does $43 \bmod 8$ change? _____ What if I add 16? _____ Or 24? _____

3 If I subtract 8 from 35 (that's 27), does $27 \bmod 8$ change? _____

4 Here's a strange one: what is **-1 mod 8**? _____ (Hint: on the mod 8 clock, start at 0 and take one step backwards.)

Patterns: Days of the Week

Cycles of 7, cycles of 5 - mod finds your place in the loop

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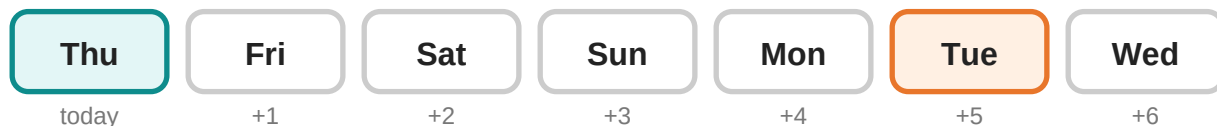
The big idea

Days repeat every 7. So 7 days from now is the same weekday, and so is 14, 21, 28... To jump ahead a lot of days, you only need the **remainder mod 7** - the full weeks don't change anything.

Example: Today is Thursday. What day is it 40 days from now?

Step 1: $40 \bmod 7$. $7 \times 5 = 35$, and $40 - 35 = 5$. (Five full weeks bring us back to Thursday, then 5 more days.)

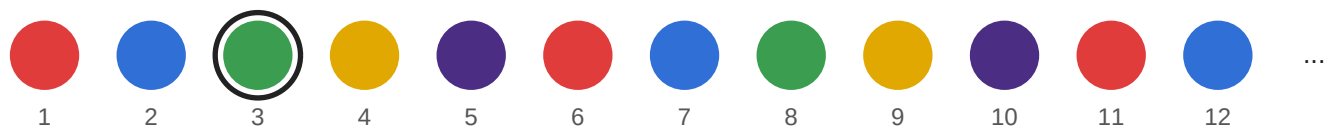
Step 2: Count 5 days forward from Thursday.



Thursday + 5 = Tuesday. The birthday is on a Tuesday!

Same trick, different loop: a 5-color bead pattern

Red, Blue, Green, Yellow, Purple, Red, Blue, Green... What color is the **43rd** bead? The pattern repeats every 5, so find $43 \bmod 5$: $5 \times 8 = 40$, $43 - 40 = 3$. The 3rd color is **Green**.



Careful with remainder 0! The 45th bead: $45 \bmod 5 = 0$. Zero doesn't mean 'no color' - it means you landed at the **end** of the pattern. Remainder 0 = the **last** item (Purple). So: remainder 1, 2, 3, 4 = 1st, 2nd, 3rd, 4th; remainder 0 = 5th.

TIP Recipe for any repeating pattern: (1) How long is the loop? (2) Find the position mod the loop length. (3) Count that many into the pattern - and remainder 0 means the last one.

Patterns: Power Cycles

The last digit of a huge power follows a tiny loop

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The secret: only the last digit matters

When you multiply, the **last digit of the answer depends only on the last digits** you multiplied. 27×3 ends in 1 because $7 \times 3 = 21$ ends in 1 - the 20 part of 27 can't touch the ones place. So the last digits of the powers of 3 go in a loop - and we can find it without ever computing the big numbers.

Example: the last digit of 3 multiplied by itself 15 times

how many 3s:	1	2	3	4	5	6	7	8
the power:	3	9	27	81	243	729	2187	6561
last digit:	3	9	7	1	3	9	7	1

Last digits: 3, 9, 7, 1, 3, 9, 7, 1, ... The loop is 3 - 9 - 7 - 1 and it repeats every 4.

$15 \bmod 4 = 3$ ($4 \times 3 = 12$, $15 - 12 = 3$). So the 15th power lands on the **3rd** spot in the loop: 3, 9, 7. The last digit is **7**. (Remainder 0 would mean the 4th spot: 1.)

Every digit has its own loop

Powers of 2:	2, 4, 8, 6	loop of 4	Powers of 3:	3, 9, 7, 1	loop of 4
Powers of 4:	4, 6	loop of 2	Powers of 7:	7, 9, 3, 1	loop of 4
Powers of 9:	9, 1	loop of 2	Powers of 5:	5	loop of 1 (always 5!)
Powers of 6:	6	loop of 1 (always 6!)			

You don't need to memorize these - just multiply last digits until the list starts over. It never takes more than 4 steps.

TIP Recipe: (1) Write the last digits until they repeat - that's your loop. (2) Take the exponent mod the loop length. (3) Count into the loop; remainder 0 means the last one.