

Distance = Rate x Time

The one formula that handles every speed problem

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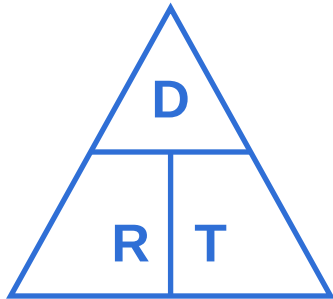
The formula

Rate is speed: how far you go in ONE unit of time (miles per hour, feet per second). **Time** is how long you travel. **Distance** is how far you end up going.

A dragon flying at **40 miles per hour** for **3 hours** covers $40 + 40 + 40 = \mathbf{120 \text{ miles}}$. That's all the formula says:

$$D = R \times T$$

Three questions, one triangle



Cover up the one you want - the other two tell you what to do.

Cover D: R and T are side by side → **multiply**: $D = R \times T$

Cover R: D is over T → **divide**: $R = D / T$

Cover T: D is over R → **divide**: $T = D / R$

Find the distance: 40 mph for 3 h → $40 \times 3 = \mathbf{120 \text{ miles}}$

Find the rate: 120 miles in 3 h → $120 / 3 = \mathbf{40 \text{ mph}}$

Find the time: 120 miles at 40 mph → $120 / 40 = \mathbf{3 \text{ hours}}$

Watch the units!

Rate in miles per **hour** means time must be in **hours**. If the time is in minutes, convert first.

Example: A monster runs at 12 mph for **30 minutes**. 30 minutes = $1/2$ hour. $D = 12 \times 1/2 = \mathbf{6 \text{ miles}}$. (NOT $12 \times 30 = 360$!)

Example: 90 miles at 60 mph. $T = 90 / 60 = 1.5 \text{ hours} = \mathbf{1 \text{ hour } 30 \text{ minutes}}$.

TIP Before you calculate, write down $D = ?$, $R = ?$, $T = ?$ with units. Two of them will be given; the third is the question.

Your Turn: $D = R \times T$

Write D , R and T first - then pick the formula

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Name: _____

$$D = R \times T$$

$$R = D / T$$

$$T = D / R$$

- 1 A dragon flies at 35 miles per hour for 4 hours. How far does it fly?

Which one is missing?

Answer: _____

- 2 A troll walks 24 miles in 6 hours. What is the troll's speed?

Rate = D / T

Answer: _____

- 3 A griffin flies at 50 mph. How long does it take to fly 200 miles?

Time = D / R

Answer: _____

- 4 A unicorn gallops at 20 mph for 45 minutes. How far does it go?

Change 45 minutes to a fraction of an hour.

Answer: _____

- 5 A wizard's broom covers 150 miles in 2 hours 30 minutes. What is its speed in mph?

2 h 30 min = 2.5 hours

- 6 A snail monster crawls 6 feet per minute. How far does it crawl in 1 hour?

How many minutes in an hour?

TIP Check: does your answer make sense? A dragon flying for 4 hours should go a LOT farther than it goes in 1 hour.

Two Travelers: Combined Rates

Toward each other, add the speeds. Chasing, subtract them.

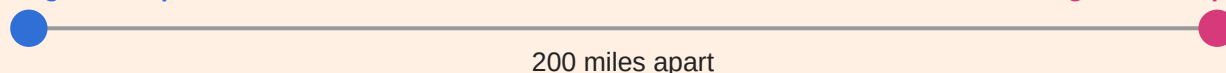
Heading toward each other: ADD the rates

A dragon flies east at **30 mph** and a griffin flies west at **20 mph**, starting **200 miles** apart. When do they meet?

Every hour the gap shrinks by $30 + 20 = 50$ miles. Treat them as ONE traveler with a combined rate of 50 mph closing a 200-mile distance:

dragon 30 mph ->

<- griffin 20 mph



$$T = D / (R1 + R2) = 200 / 50 = 4 \text{ hours}$$

Chasing (same direction): SUBTRACT the rates

A troll has a **60-mile** head start walking at **10 mph**. A dragon chases at **40 mph**. When does the dragon catch up?

Every hour the dragon gains $40 - 10 = 30$ miles on the troll. The gap to close is 60 miles:

dragon 40 mph ->

troll 10 mph ->



$$T = \text{gap} / (R1 - R2) = 60 / 30 = 2 \text{ hours}$$

Round trips: add the distances AND the times

A dragon flies 60 miles to a cave at 30 mph (2 hours) and flies back at 20 mph (3 hours). What's its average speed?

Trap: it is NOT $(30 + 20) / 2 = 25$. Average speed = **total distance / total time** = $(60 + 60) / (2 + 3) = 120 / 5 = 24$ mph. The slow leg takes longer, so it drags the average down.

TIP Ask: is the gap shrinking because BOTH are moving toward it (add) or because one is faster than the other (subtract)? Then it's just $D = R \times T$ with the combined rate.

Your Turn: Two Travelers

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Combine the rates first, then $D = R \times T$

Name: _____

Toward each other: $R1 + R2$

Chasing: $R1 - R2$

Average speed = total distance / total time

- 1 Two monsters start 150 miles apart and walk toward each other, one at 20 mph and one at 30 mph. In how many hours do they meet?

Add the rates.

Answer: _____

- 2 A unicorn and a dragon leave the same castle in opposite directions, at 25 mph and 35 mph. How far apart are they after 3 hours?

Opposite directions: the gap GROWS by the combined rate.

Answer: _____

- 3 A goblin has a 40-mile head start and runs at 8 mph. A wolf chases at 18 mph. How long until the wolf catches the goblin?

Subtract the rates.

Answer: _____

TIP Draw a line with the two travelers on it and label the gap. The picture tells you whether to add or subtract.

Your Turn: Two Travelers

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Combine the rates first, then $D = R \times T$

Name: _____

Toward each other: $R1 + R2$

Chasing: $R1 - R2$

Average speed = total distance / total time

- 4 Two knights ride toward each other from towns 90 miles apart. One rides at 12 mph and the other at 18 mph. How far has the faster knight gone when they meet?

Find the time first, then $D = R \times T$ for the faster one.

Answer: _____

- 5 A dragon flies 100 miles to an island at 50 mph and flies back at 25 mph. What is its average speed for the whole trip?

Total distance / total time.

Answer: _____

- 6 A bat flies 30 minutes at 20 mph, then 30 minutes at 40 mph. What is its average speed?

Is this one a trap too?

Answer: _____

TIP Draw a line with the two travelers on it and label the gap. The picture tells you whether to add or subtract.

Other Rates

Same idea, different things: any 'something PER something'

A rate is anything 'per'

Speed is miles **per** hour. But a rate can be cookies per batch, dollars per ticket, gallons per minute, pages per day... The formula is the same, just with new names:

$$\text{Total} = \text{Rate} \times \text{How many}$$

Examples

A dragon eats 4 sheep per day. How many in a week?

$$\text{Total} = 4 \text{ per day} \times 7 \text{ days} = \mathbf{28 \text{ sheep}}$$

Tickets cost \$6 each. 9 monsters buy tickets. Total cost?

$$\text{Total} = \$6 \text{ per ticket} \times 9 \text{ tickets} = \mathbf{\$54}$$

A fountain fills 5 gallons per minute. How long to fill 60 gallons?

$$\text{How many} = 60 / 5 = \mathbf{12 \text{ minutes}}$$

A wizard reads 240 pages in 8 days. Pages per day?

$$\text{Rate} = 240 / 8 = \mathbf{30 \text{ pages per day}}$$

Unit rate: get it down to ONE

If a problem gives you a rate for a bunch at once, find the rate for **one** first - that's the **unit rate**.

Example: 3 potions cost \$12. How much for 7 potions? Unit rate: $\$12 / 3 = \mathbf{\$4 \text{ per potion}}$. Then $7 \times \$4 = \mathbf{\$28}$.

Example: A dragon flies 90 miles in 2 hours. How far in 5 hours? $90 / 2 = 45 \text{ mph}$. $45 \times 5 = \mathbf{225 \text{ miles}}$.

TIP Say the rate out loud with 'per' in it. If you can say '___ per ___', you know which number to multiply or divide by.

Your Turn: Other Rates

Find the rate, then multiply or divide

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Name: _____

Total = Rate x How many

Rate = Total / How many

How many = Total / Rate

- 1** A goblin earns \$9 per hour. How many hours must it work to earn \$63?

Answer: _____

- 2** 4 magic feathers cost \$20. How much do 11 feathers cost?

Unit rate first!

Answer: _____

- 3** A troll stacks 36 boulders in 3 hours. At that rate, how many boulders in 7 hours?

Boulders per hour first.

Answer: _____

TIP Unit rate first: get the rate down to ONE hour, ONE potion, ONE day - then scale it up.

Dimensional Analysis

Converting units by multiplying by 1

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The trick: a fraction that equals 1

12 inches and 1 foot are the **same length**. So the fraction **12 in / 1 ft** equals **1**, and so does **1 ft / 12 in**. Multiplying by 1 doesn't change an amount - it just changes the **units**. Pick the version that **cancels the unit you have** and leaves the unit you want.

Example: 60 inches to feet

$$60 \cancel{\text{in}} \times \frac{1 \text{ ft}}{12 \cancel{\text{in}}} = 5 \text{ ft}$$

We have inches and want feet, so **inches goes on the bottom** of the fraction. The **in** on top and the **in** on the bottom cancel out, like a 3 on top and a 3 on the bottom. Only **feet** are left. Then do the numbers: $60 \times 1 / 12 = 5$. The 12 ended up in the denominator - that's a divide!
Going the other way: $5 \text{ ft} \times (12 \text{ in} / 1 \text{ ft}) = 5 \times 12 = 60 \text{ in}$. Now the feet cancel and the 12 is on top.

Chain them: more than one step

2 miles to feet: $2 \text{ mi} \times (5280 \text{ ft} / 1 \text{ mi}) = 10,560 \text{ ft}$

3 hours to seconds: $3 \text{ h} \times (60 \text{ min} / 1 \text{ h}) \times (60 \text{ s} / 1 \text{ min}) = 3 \times 60 \times 60 = 10,800 \text{ s}$ - hours cancel, then minutes cancel.

30 mph to miles per minute: $30 \text{ mi/h} \times (1 \text{ h} / 60 \text{ min}) = 30 / 60 = 1/2 \text{ mile per minute}$.

Conversions to know

Length: $1 \text{ ft} = 12 \text{ in}$ | $1 \text{ yd} = 3 \text{ ft}$ | $1 \text{ mi} = 5280 \text{ ft}$ | 1 mi is about 1.6 km | $1 \text{ m} = 100 \text{ cm}$ | $1 \text{ km} = 1000 \text{ m}$

Time: $1 \text{ h} = 60 \text{ min}$ | $1 \text{ min} = 60 \text{ s}$ | $1 \text{ day} = 24 \text{ h}$ | $1 \text{ week} = 7 \text{ days}$

Other: $1 \text{ lb} = 16 \text{ oz}$ | $1 \text{ gal} = 4 \text{ qt}$ | $1 \text{ qt} = 2 \text{ pt}$ | $1 \text{ dozen} = 12$

TIP The unit you want to GET RID OF goes on the bottom of the fraction. The unit you WANT goes on top.

Your Turn: Dimensional Analysis

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Write the fraction, cross out the units, multiply

Name: _____

1 ft = 12 in 1 yd = 3 ft 1 mi = 5280 ft 1 mi = 1.6 km 1 h = 60 min

1 Convert 7 feet to inches.

$7 \text{ ft} \times (12 \text{ in} / 1 \text{ ft})$

Answer: _____

2 Convert 15 yards to feet.

Answer: _____

3 Convert 96 inches to feet.

Which unit goes on the bottom?

Answer: _____

4 Convert 3 miles to feet.

$1 \text{ mi} = 5280 \text{ ft}$

Answer: _____

5 Convert 10 miles to kilometers (use 1 mi = 1.6 km).

6 A dragon flies 2 miles per minute. How many miles per hour is that?

Two steps? No - just one: minutes to hours.

TIP After you cross out the units, the only unit left should be the one the question asked for. If not, flip a fraction!

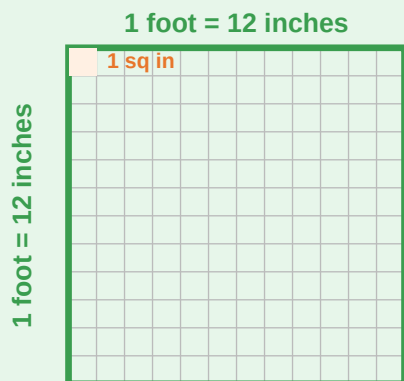
Converting Units with Area

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Why 1 square foot is NOT 12 square inches

The trap

A square foot is a square that's 1 foot on each side. That's 12 inches by 12 inches. How many square inches fit inside?



Count: 12 rows of 12 little squares = $12 \times 12 = 144$ square inches.

1 sq ft = 144 sq in, not 12. The conversion number gets **squared** because area has **two** dimensions - you convert the length AND the width.

Same idea: 1 sq yd = $3 \times 3 = 9$ sq ft. 1 sq m = $100 \times 100 = 10,000$ sq cm.

Dimensional analysis still works - use the fraction twice

3 sq ft to sq in: $3 \text{ ft}^2 \times (12 \text{ in} / 1 \text{ ft}) \times (12 \text{ in} / 1 \text{ ft}) = 3 \times 144 = 432 \text{ sq in}$

One fraction cancels one 'ft', the second fraction cancels the other. Two ft's need two fractions!

Shortcut: square the conversion number once and remember it: 1 sq ft = 144 sq in, 1 sq yd = 9 sq ft.

Going smaller to bigger: 54 sq ft to sq yd = $54 / 9 = 6$ sq yd.

A dragon's rug

A dragon's rug is **2 yards by 3 yards**. What's its area in square feet?

Way 1: area = $2 \times 3 = 6$ sq yd, then $6 \times 9 = 54$ sq ft. **Way 2:** convert sides first: $6 \text{ ft} \times 9 \text{ ft} = 54$ sq ft. Same answer!

TIP Length: use the conversion number once. Area: use it TWICE (square it). Volume would be three times!

Your Turn: Area Units

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Square the conversion number!

Name: _____

1 sq ft = 144 sq in

1 sq yd = 9 sq ft

1 sq m = 10,000 sq cm

- 1 How many square inches are in 2 square feet?

1 sq ft = 144 sq in

Answer: _____

- 2 How many square feet are in 4 square yards?

1 sq yd = ____ sq ft

Answer: _____

- 3 A monster's doormat is 1 foot by 2 feet. What is its area in square inches?

Two ways: convert first, or convert after.

Answer: _____

- 4 A dragon's cave floor is 36 square feet. How many square yards is that?

Divide.

Answer: _____

- 5 A wizard's garden is 5 yards by 4 yards. How many square feet?

- 6 A tile is 6 inches by 6 inches. How many tiles cover 1 square foot?

Draw it!

TIP If you're not sure, draw the square and count the little squares inside. The picture never lies.