

Adding Fractions: Same Denominator

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Same-size pieces: just count them

What the two numbers mean

The **bottom** number (the **denominator**) tells you how big the pieces are - fifths are pieces of a whole cut into 5. The **top** number (the **numerator**) tells you how many of those pieces you have.

$\frac{2}{5}$ = two pieces, each one-fifth of the whole.



Think of the denominator as the NAME of the piece (fifths) and the numerator as the COUNT (two). '2 fifths' works just like '2 apples'.

The rule: add the tops, KEEP the bottom

If the pieces are the same size, adding is just counting. 1 fifth + 2 fifths = 3 fifths. The pieces are still fifths!



+



=



$$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$$

Never add the bottoms. $\frac{1}{5} + \frac{2}{5}$ is NOT $\frac{3}{10}$ - the pieces didn't get smaller, you just have more of them.

Simplify at the end: $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ (divide top and bottom by 2). **More than a whole:** $\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = 1$ and $\frac{2}{4} = 1$ and $\frac{1}{2}$.

Your turn

1. $\frac{3}{8} + \frac{2}{8} =$ _____
2. $\frac{1}{7} + \frac{4}{7} =$ _____
3. $\frac{2}{9} + \frac{5}{9} =$ _____
4. $\frac{1}{4} + \frac{1}{4} =$ _____ (simplify!)
5. $\frac{5}{6} + \frac{4}{6} =$ _____ (more than 1!)

TIP The denominator is a label, not a number to add. 2 apples + 3 apples = 5 apples, not 5 apple-apples.

Adding Fractions: Different Denominators

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Make the pieces the same size first

Why you can't just add

$1/2 + 1/3$: a half and a third are different-sized pieces. '1 half + 1 third' is like '1 apple + 1 orange' - you can't call it 2 of anything until you find a name that fits both. We need a piece size that halves AND thirds can both be cut into.

Step 1: Find the common denominator = the LCM of the bottoms

List multiples of each bottom until they match - that's the LCM (from the factors guide!).

Multiples of 2: 2, 4, **6**, 8, 10 ... **Multiples of 3:** 3, **6**, 9, 12 ... → common denominator is **6**.

Quick cases:

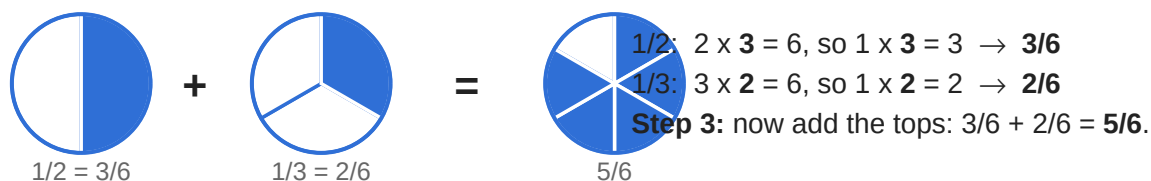
One bottom divides the other ($1/4 + 1/8$): use the bigger one. LCM = **8**.

Bottoms share nothing ($1/3 + 1/5$): multiply them. LCM = **15**.

Otherwise ($1/6 + 1/4$): list multiples. 6, **12** ... 4, 8, **12**. LCM = **12**, not 24.

Step 2: Convert each fraction - multiply top AND bottom by the same number

Ask: 'What do I multiply the bottom by to get 6?' Then multiply the top by the **same** number. This doesn't change the amount - cutting each half into 3 pieces gives 3 sixths, same amount of pie.



Find the LCM -> convert each fraction -> add the tops -> simplify

TIP Whatever you do to the bottom, do to the top. Multiplying both by 3 is multiplying by $3/3 = 1$, so the value never changes.

Subtracting Fractions

Exactly like adding - match the pieces, then subtract the tops

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Same bottoms: subtract the tops, keep the bottom

$$\frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$

5 sevenths take away 2 sevenths leaves 3 sevenths. The 7 stays - the pieces didn't change size.

Different bottoms: common denominator first

Example: $\frac{3}{4} - \frac{1}{6}$. LCM of 4 and 6 is **12**.

$\frac{3}{4} = \frac{9}{12}$ (x3 top and bottom) $\frac{1}{6} = \frac{2}{12}$ (x2 top and bottom)

$$\frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

Sanity check: $\frac{3}{4}$ is a bit less than 1, $\frac{1}{6}$ is small, so the answer should be a bit more than $\frac{1}{2}$. $\frac{7}{12}$ is just over $\frac{6}{12} = \frac{1}{2}$. Good!

Subtracting from a whole number

Example: $1 - \frac{3}{8}$. Write the 1 as eighths: $1 = \frac{8}{8}$. Then $\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$.

Example: $3 - \frac{1}{4}$. Borrow one whole as quarters: $3 = 2$ and $\frac{4}{4}$. Then 2 and $\frac{4}{4} - \frac{1}{4} = 2$ and $\frac{3}{4}$.

This shows up in work problems: 'how much of the job is LEFT?' is always 1 - (the part done).

Your turn

1. $\frac{6}{9} - \frac{2}{9} =$ _____

2. $\frac{1}{2} - \frac{1}{4} =$ _____

3. $\frac{5}{6} - \frac{1}{3} =$ _____

4. $1 - \frac{2}{5} =$ _____

5. $\frac{3}{4} - \frac{2}{3} =$ _____

6. $2 - \frac{1}{3} =$ _____

TIP Bigger minus smaller: check that your answer is positive and smaller than the first fraction.

Your Turn: Adding and Subtracting

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Match the bottoms first - then add or subtract the tops

Name: _____

1 $\frac{2}{7} + \frac{3}{7}$

= _____

2 $\frac{5}{9} - \frac{2}{9}$

= _____

3 $\frac{1}{3} + \frac{1}{6}$

= _____

4 $\frac{3}{4} - \frac{1}{8}$

= _____

5 $\frac{1}{2} + \frac{1}{5}$

= _____

6 $\frac{2}{3} - \frac{1}{4}$

= _____

7 $\frac{1}{6} + \frac{3}{8}$

= _____

8 $1 - \frac{5}{12}$

= _____

9 $\frac{3}{10} + \frac{2}{5}$

= _____

10 $\frac{5}{6} - \frac{3}{4}$

= _____

11 $\frac{1}{2} + \frac{1}{3} + \frac{1}{6}$

= _____

12 $2 - \frac{2}{3}$

= _____

TIP Simplify every answer. If the top is bigger than the bottom, turn it into a mixed number.

Multiplying Fractions

The easy one: top times top, bottom times bottom

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The rule - no common denominator needed!

$$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$$

Multiply straight across. Multiplying is the ONLY operation where the bottoms don't have to match.

What does it mean? 'Of'

$\frac{1}{2} \times \frac{1}{3}$ means **one half OF one third**. Take a third of a bar, then take half of THAT piece - you get $\frac{1}{6}$ of the whole bar.



dark = half of the first third = $\frac{1}{6}$ of the bar

Whole number x fraction: $4 \times \frac{2}{3} = \frac{8}{3} = 2 \text{ and } \frac{2}{3}$. **Fraction OF an amount:** $\frac{3}{4}$ of 20 = $\frac{60}{4} = 15$.

Cancelling: simplify BEFORE you multiply

Since everything on top gets multiplied together and everything on the bottom gets multiplied together, you can divide **any** top number and **any** bottom number by a common factor before you multiply - even if they're in different fractions. It's the same as simplifying at the end, but the numbers stay small.

Example: $\frac{3}{8} \times \frac{4}{9}$

Look for a top and a bottom that share a factor.

3 (top) and **9** (bottom) share 3: $3 \div 3 = 1$, $9 \div 3 = 3$

4 (top) and **8** (bottom) share 4: $4 \div 4 = 1$, $8 \div 4 = 2$

Now it's $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$. Done - and no big numbers.

$$\begin{array}{c} 1 \\ \cancel{3} \\ \hline \cancel{8} \\ 2 \end{array} \times \begin{array}{c} 1 \\ \cancel{4} \\ \hline \cancel{9} \\ 3 \end{array}$$

cross out, write the new number

Without cancelling: $3 \times 4 = 12$, $8 \times 9 = 72$, then simplify $\frac{12}{72}$ by the GCF $12 = \frac{1}{6}$. Same answer, more work.

Rule: you may cancel a TOP with a BOTTOM. Never a top with a top.

TIP Multiplying by a fraction less than 1 makes things SMALLER. That's why 'half of' means 'x $\frac{1}{2}$ '.

Your Turn: Multiplying

Cancel first when you can - then straight across

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Name: _____

1 $\frac{1}{2} \times \frac{1}{4}$

= _____

2 $\frac{2}{3} \times \frac{3}{5}$

= _____

3 $\frac{3}{4} \times \frac{2}{9}$

= _____

4 $5 \times \frac{3}{4}$

= _____

5 $\frac{4}{7} \times \frac{7}{8}$

= _____

6 $\frac{5}{6} \times \frac{3}{10}$

= _____

7 $\frac{2}{3}$ of 18

= _____

8 $\frac{3}{8} \times \frac{4}{9}$

= _____

9 $\frac{9}{10} \times \frac{5}{6}$

= _____

10 $\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

= _____

11 $\frac{7}{12} \times \frac{6}{7}$

= _____

12 $1 \text{ and } \frac{1}{2} \times \frac{2}{3}$

= _____

TIP Last one: turn the mixed number into an improper fraction first ($1 \text{ and } \frac{1}{2} = \frac{3}{2}$).

What Is a Reciprocal?

Flip it upside down

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The definition

The **reciprocal** of a fraction is the fraction **flipped**: swap the top and bottom. The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. The reciprocal of 5 (which is $\frac{5}{1}$) is $\frac{1}{5}$.

$$\frac{2}{3} \text{ flip } \rightarrow \frac{3}{2}$$

$$\frac{1}{4} \text{ flip } \rightarrow \frac{4}{1} = 4$$

The magic: a number times its reciprocal is always 1

$$\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$$

$$5 \times \frac{1}{5} = \frac{5}{5} = 1$$

That's why reciprocals matter: multiplying by the reciprocal **undoes** multiplying by the number. It's the fraction version of 'the opposite of x5 is divide by 5'.

Reciprocals in work problems (you've seen this!)

A dragon does $\frac{1}{4}$ of the job per hour. How long for the whole job? **4 hours** - the reciprocal of $\frac{1}{4}$. Together they do $\frac{2}{5}$ of the job per hour. Whole job: $\frac{5}{2} = 2 \text{ and } \frac{1}{2} \text{ hours}$ - the reciprocal of $\frac{2}{5}$. 'Flip the share to get the time' is exactly 'take the reciprocal'.

Your turn

1. Reciprocal of $\frac{3}{7}$: _____
2. Reciprocal of 8: _____
3. Reciprocal of $\frac{1}{9}$: _____
4. Reciprocal of 2 and $\frac{1}{2}$ (hint: write it as $\frac{5}{2}$ first): _____
5. $\frac{4}{5} \times$ _____ = 1

TIP A mixed number must become an improper fraction BEFORE you flip it. $1 \text{ and } \frac{1}{2} = \frac{3}{2}$, so its reciprocal is $\frac{2}{3}$ (not 1 and $\frac{2}{1}$!).

Dividing Fractions

Keep, Change, Flip

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The rule: dividing by a fraction = multiplying by its reciprocal

KEEP the first fraction. **CHANGE** the division sign to multiplication. **FLIP** the second fraction. Then multiply across.

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8} = 1 \text{ and } 7/8$$

keep 3/4 change / to x flip 2/5 into 5/2 multiply

Why does flipping work? 'How many fit?'

6 / 2 asks 'how many 2s fit in 6?' Answer: 3.

6 / 1/2 asks 'how many **halves** fit in 6?' Each whole has 2 halves, so $6 \times 2 = 12$. See? Dividing by $1/2$ is the same as multiplying by 2 - the reciprocal!



6 wholes, each cut in half = 12 halves. So $6 \div (1/2) = 12$.

Dividing a fraction by a whole number: $3/4 \div 2 = 3/4 \times 1/2 = 3/8$. (Half of three-quarters.)

Sanity check

Dividing by a fraction **less than 1** makes the answer **BIGGER** (how many small pieces fit? lots!). Dividing by a number bigger than 1 makes it smaller. $3/4 \div 2/5 = 1 \text{ and } 7/8$ - bigger than $3/4$. Makes sense.

Your turn

1. $1/2 \div 1/4 =$ _____ 2. $2/3 \div 4/5 =$ _____ 3. $5 \div 1/3 =$ _____ 4. $3/8 \div 3 =$ _____

TIP Only flip the **SECOND** fraction - the one you're dividing **BY**. Never flip the first.

Your Turn: Dividing

Keep, Change, Flip - then cancel and multiply

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Name: _____

1 $\frac{1}{2} \div \frac{1}{4}$

= _____

2 $\frac{2}{3} \div \frac{4}{5}$

= _____

3 $\frac{3}{4} \div \frac{3}{8}$

= _____

4 $5 \div \frac{1}{3}$

= _____

5 $\frac{3}{8} \div 3$

= _____

6 $\frac{5}{6} \div \frac{5}{12}$

= _____

7 $\frac{7}{10} \div \frac{7}{5}$

= _____

8 $1 \div \frac{2}{7}$

= _____

9 $\frac{4}{9} \div \frac{2}{3}$

= _____

10 $\frac{1}{2} \div 4$

= _____

11 $\frac{9}{8} \div \frac{3}{4}$

= _____

12 $2 \text{ and } \frac{1}{2} \div \frac{1}{2}$

= _____

TIP Sanity check: dividing by a fraction smaller than 1 should give a BIGGER answer.

Nested Fractions

A fraction inside a fraction is just a division in disguise

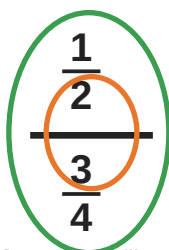
The big fraction bar means DIVIDE

A fraction bar is a division sign: $6/3$ means $6 / 3 = 2$. So a fraction with fractions on top and bottom just means **top fraction divided by bottom fraction**. Then it's Keep, Change, Flip:

$$\frac{\frac{1}{2}}{\frac{3}{4}} = \frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \times \frac{4}{3} = \frac{4}{6} = \frac{2}{3}$$

The Donut Method: 'Do the donut'

A faster way to see the same thing. Draw a loop from the **outside** numbers (very top and very bottom) - they multiply to make the new **top**. Then loop the **inside** numbers (the two in the middle) - they multiply to make the new **bottom**.



outside loop: $1 \times 4 = 4 \rightarrow$ new top

inside loop: $2 \times 3 = 6 \rightarrow$ new bottom

$$= 4 / 6 = 2 / 3$$

Why it works: flipping $3/4$ and multiplying gives 1×4 on top and 2×3 on the bottom - exactly the outside and inside numbers. *Outside over inside. Do the donut!*

When the top or bottom has its own math: inside out

Simplify the top and the bottom **separately first**, then do the donut.

($1/2 + 1/3$) over ($1/6$): top = $3/6 + 2/6 = 5/6$; bottom = $1/6$. Donut: outside $5 \times 6 = 30$, inside $6 \times 1 = 6$. $30/6 = 5$.

1 over ($1/4 + 1/4$): bottom = $1/2$, so it's $1 / (1/2) = 2$. *This is exactly the 'flip' in work problems.*

TIP Top first, bottom second, donut last. A whole number counts as 'over 1' ($2 = 2/1$) so it has an outside and an inside too.

Your Turn: Nested Fractions

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Simplify top, simplify bottom, do the donut

Name: _____

1

$$\frac{\frac{2}{3}}{\frac{1}{3}}$$

= _____

2

$$\frac{\frac{3}{4}}{\frac{1}{2}}$$

= _____

3

$$\frac{\frac{1}{5}}{\frac{2}{5}}$$

= _____

4

$$\frac{\frac{5}{8}}{\frac{5}{6}}$$

= _____

5

$$\frac{2}{\frac{1}{4}}$$

= _____

6

$$\frac{\frac{3}{10}}{6}$$

= _____

7

$$\frac{\frac{1}{4} + \frac{1}{4}}{\frac{1}{2}}$$

= _____

8

$$\frac{\frac{2}{3}}{\frac{1}{2} + \frac{1}{6}}$$

= _____

9

$$\frac{1}{\frac{1}{3} + \frac{1}{6}}$$

= _____

TIP Problems 7, 8 and 9: add inside the top or bottom first, THEN do the donut.

Your Turn: Mixed Practice

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Which operation is it? Then follow the rule

Name: _____

1 $\frac{1}{3} + \frac{1}{4}$

= _____

2 $\frac{5}{8} - \frac{1}{4}$

= _____

3 $\frac{2}{5} \times \frac{5}{6}$

= _____

4 $\frac{3}{4} \div \frac{1}{8}$

= _____

5 $1 - \frac{3}{7}$

= _____

6 $6 \times \frac{2}{9}$

= _____

TIP Box colors match the pages: blue add, teal subtract, green multiply, purple reciprocal, orange divide, pink nested.

Your Turn: Mixed Practice

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Which operation is it? Then follow the rule

Name: _____

7 $(\frac{1}{2})$ over $(\frac{1}{6})$

= _____

8 $\frac{2}{3} + \frac{1}{6} - \frac{1}{2}$

= _____

9 $\frac{4}{5} \div 2$

= _____

10 1 over $(\frac{1}{3} + \frac{1}{6})$

= _____

11 Reciprocal of 3 and $\frac{1}{3}$

= _____

12 $\frac{1}{2}$ of $\frac{3}{4}$

= _____

TIP Box colors match the pages: blue add, teal subtract, green multiply, purple reciprocal, orange divide, pink nested.

Fractions Cheat Sheet

Everything on one page

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ADD

Match the bottoms (LCM), convert, add the tops, keep the bottom

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}$$

SUBTRACT

Match the bottoms, convert, subtract the tops, keep the bottom

From a whole: $1 = \frac{8}{8}$, $3 = 2$ and $\frac{4}{4}$

$$\frac{3}{4} - \frac{1}{6} = \frac{9}{12} - \frac{2}{12} = \frac{7}{12}$$

MULTIPLY

Top x top, bottom x bottom. No common denominator!
'of' means multiply. Cancel first to keep numbers small.

$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

RECIPROCAL

Flip it. A number times its reciprocal = 1

Mixed number -> improper fraction first, then flip

$$\frac{2}{3} \rightarrow \frac{3}{2} \quad 5 \rightarrow \frac{1}{5}$$

DIVIDE

Keep, Change, Flip: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

Dividing by a fraction < 1 makes it BIGGER

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8}$$

NESTED

The big bar means divide: simplify top, simplify bottom,
then top / bottom = top x (flipped bottom)

$$(\frac{1}{2}) / (\frac{3}{4}) = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$$

Always simplify at the end: divide top and bottom by their GCF.