

Factors & Multiples: The Big Words

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Learn these and the rest is easy!

FACTOR

A factor is a number that divides evenly into another number (nothing left over).

Example: The factors of **12** are **1, 2, 3, 4, 6, 12**. Each one goes into 12 with no remainder.

Factors are FEW. They never go above the number itself.

MULTIPLE

A multiple is what you get when you multiply a number by 1, 2, 3, 4, ...

Example: The multiples of **4** are **4, 8, 12, 16, 20, 24, ...** (just skip-count by 4!)

Multiples are MANY. They go on forever.

PRIME NUMBER

A prime number has exactly two factors: 1 and itself.

Example: **2, 3, 5, 7, 11, 13, 17, 19, 23, 29** are the first ten primes.

1 is NOT prime (it has only one factor). 2 is the only even prime.

COMPOSITE NUMBER

A composite number has more than two factors. It can be broken into smaller pieces.

Example: **12** is composite: $12 = 3 \times 4 = 2 \times 2 \times 3$.

Every whole number bigger than 1 is either prime or composite.

GREATEST COMMON FACTOR (GCF)

The BIGGEST number that is a factor of two (or more) numbers.

Example: GCF of **12** and **18** is **6**, because 6 is the biggest number that divides both.

Also called the GCD (Greatest Common Divisor). Same thing!

LEAST COMMON MULTIPLE (LCM)

The SMALLEST number that is a multiple of two (or more) numbers.

Example: LCM of **4** and **6** is **12**, because 12 is the first number in both skip-count lists.

Think: "When do they first land on the same number?"

TIP **Factors are few, multiples are many.** If you forget which is which, remember: the factors of 12 stop at 12, but the multiples of 12 (12, 24, 36, ...) never stop.

Your Turn: The Big Words

Fill in, circle, and match

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Name: _____

Part A: Find the GCF and LCM of 12 and 18 on the picture

Factors of 12: 1 2 3 4 6 12

Factors of 18: 1 2 3 6 9 18

1. **Circle** every number that is in BOTH lists. 2. The biggest one you circled is the GCF. **GCF of 12 and 18 = _____**

Multiples of 12: 12 24 36 48 60 72 ...

Multiples of 18: 18 36 54 72 90 108 ...

3. **Draw a box** around every number that is in BOTH lists. 4. The smallest one you boxed is the LCM. **LCM of 12 and 18 = _____**

5. Check the shortcut: $\text{GCF} \times \text{LCM} = \underline{\hspace{2cm}}$ and $12 \times 18 = \underline{\hspace{2cm}}$ Are they the same?

Part B: Draw a line from each word to its example

FACTOR

MULTIPLE

PRIME NUMBER

COMPOSITE NUMBER

GCF

LCM

36 is this for the pair 12 and 18

4 is this for 12, because $12 \div 4$ comes out even

15, because it can be split into 3×5

6 is this for the pair 12 and 18

40 is this for 8, because $8 \times 5 = 40$

7, because only 1 and 7 divide it

Bonus: write your own example for MULTIPLE here: _____

How to Make a Factor Tree

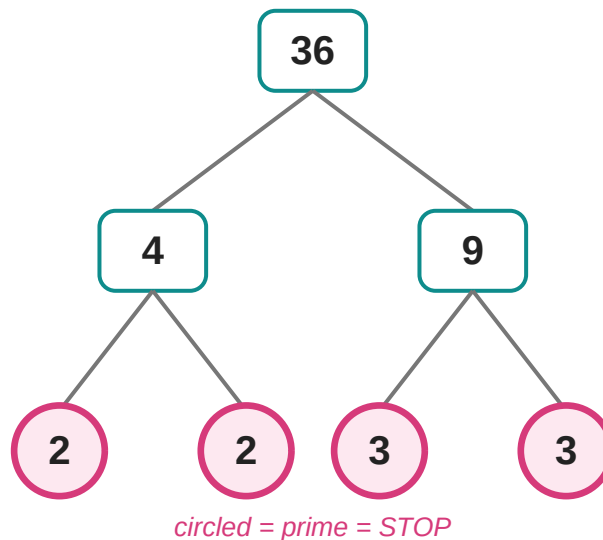
Break a number into its prime 'ingredients'

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The Steps

- 1 Write the number at the top.
- 2 Split it into ANY two numbers that multiply to make it. (Not 1 x the number!)
- 3 Look at each branch. Is it prime? **Circle it** - it's done!
- 4 Not prime? Split it again.
- 5 Keep going until every branch ends in a circled prime.
- 6 Write the number as a product of all the circled primes.

Let's do 36!



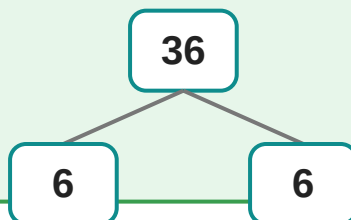
$$36 = 2 \times 2 \times 3 \times 3$$

(Read the circles left to right!)

$$\text{With exponents: } 36 = 2^2 \times 3^2$$

The exponent counts how many times a prime repeats.

Does it matter how you split? No! Try starting $36 = 6 \times 6$:



Each 6 splits into 2×3 , so you get **2, 3, 2, 3** - the SAME primes, just in a different order. Either way, $36 = 2^2 \times 3^2$. Every number has exactly one set of prime ingredients - that's why factor trees are so useful for GCF and LCM later!

TIP A quick way to split: if the number is even, start with 2. If its digits add up to a multiple of 3, start with 3. Ends in 0 or 5? Start with 5.

Your Turn: Factor Trees

Grow a tree for each number, circle the primes

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Name: _____

Start by splitting the number into any two factors (not 1 x itself). Keep splitting until every branch ends in a prime. Circle the primes, then write the number as a product of primes - plain and with exponents.

60

35

48

(keep going down!)

(keep going down!)

(keep going down!)

60 = _____

35 = _____

48 = _____

With exponents:

With exponents:

With exponents:

60 = _____

35 = _____

48 = _____

TIP Stuck? 60 ends in 0, so start with 10 x 6. 35 ends in 5, so start with 5. 48 is even, so start with 2 x 24 (or 6 x 8).

Finding ALL the Factor Pairs

Count up from 1 and stop when the numbers meet

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The Steps

- 1 Make two columns. Start with **1** on the left. Its partner (the number itself) goes on the right.
- 2 Try **2**. Does it divide evenly? Yes: write 2 on the left and its partner on the right. No: skip it.
- 3 Try **3**, then **4**, then **5**... one number at a time, going UP.
- 4 Watch the two columns. The left goes up, the right comes down.
- 5 **STOP** when the next number you'd try is already in the right column. The numbers have **met in the middle!**
- 6 Read all the numbers in both columns - those are ALL the factors.

Let's find the factors of 30!

small		big
1		30
	$1 \times 30 = 30$	
2		15
	$2 \times 15 = 30$	
3		10
	$3 \times 10 = 30$	
4		-
	4? $30 / 4$ isn't whole. Skip!	
5		6
	$5 \times 6 = 30$	

Next would be 6 - but 6 is already in the big column. They MET! STOP.

All the factors of 30:

1, 2, 3, 5, 6, 10, 15, 30

That's 8 factors. 4 pairs $\times 2 = 8$!

Special case: square numbers

For **16**: 1 and 16, 2 and 8, 4 and **4**. The columns meet on the SAME number! Count that 4 only once, so 16 has **5** factors: 1, 2, 4, 8, 16. (Square numbers are the only ones with an odd number of factors.)

TIP Pairs always meet somewhere in the middle, so you never have to check all the way up to 30. Once the small column catches the big column, you're done!

Your Turn: Factor Pairs

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Two columns, count up, stop when they meet

Name: _____

Start with 1 on the left and the number on the right. Try 2, 3, 4... one at a time. Write each pair that works, skip the ones that don't, and STOP when the next number is already in the right column.

Factors of 36

small

big

1

36

Factors of 40

small

big

1

40

Factors of 28

small

big

1

28

All the factors of 36:

How many factors? _____

All the factors of 40:

How many factors? _____

All the factors of 28:

How many factors? _____

Think: one of these numbers is a square number. Which one? _____ How could you tell from your table?

TIP Check: every pair should multiply to the number. If a pair doesn't, you've found a mistake!

Greatest Common Factor (GCF)

The biggest number that fits into BOTH

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Method 1: List the factors

- 1 List ALL the factors of each number (use the two-column method!).
- 2 Circle the factors that appear in BOTH lists. These are the **common factors**.
- 3 The biggest circled number is the **GCF**.

Example: GCF of 18 and 24

18: 1 2 3 6 9 18

24: 1 2 3 4 6 8 12 24

Common factors: 1, 2, 3, 6

GCF = 6

Check: $18 \div 6 = 3$ and $24 \div 6 = 4$. No remainders, and nothing bigger works. The GCF can never be bigger than the smaller number.

Method 2: Factor trees (great for BIG numbers)

Listing every factor of 96 and 144 would take forever. Factor trees make it quick - turn to the **next page** for the step-by-step picture.

Word-problem clue words for GCF

'identical groups', 'split evenly', 'the greatest number of bags/rows/chests', 'nothing left over'. You are DIVIDING things up and want the biggest group size or most groups.

TIP If one number is a factor of the other (like 6 and 18), the GCF is just the smaller number. If the only common factor is 1 (like 8 and 15), the GCF is 1.

GCF with Factor Trees

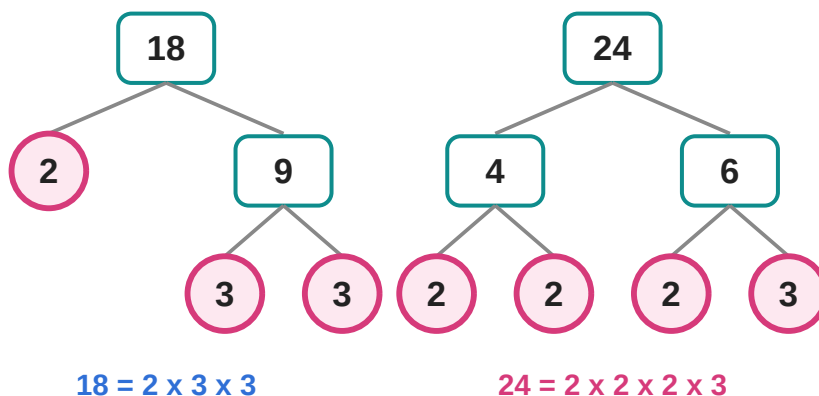
Find the primes they SHARE

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The Steps

- 1 Make a factor tree for EACH number.
- 2 Write each number's list of primes.
- 3 Write one list above the other, and **line up matching primes** in columns: 2s under 2s, 3s under 3s. Leave a gap when one number runs out.
- 4 Circle each column that has a prime in BOTH rows.
- 5 Multiply one prime from each circled column. That's the GCF!

Example: GCF of 18 and 24



Line up the primes

$$\begin{array}{rccccccc} 18 = & 2 & & \times & & 3 & \times & 3 \\ & \downarrow & & & & \downarrow & & \\ 24 = & 2 & \times & 2 & \times & 2 & \times & 3 \\ \hline \text{GCF} = & 2 & & \times & & 3 & & \end{array}$$

Two circled columns: a 2 and a 3. $\text{GCF} = 2 \times 3 = 6$

$$\text{GCF} = 2 \times 3 = 6$$

Faster with exponents? Once this picture makes sense, the **next page** shows the same problem written with exponents - no lining up needed.

TIP Why circles? A number that divides 18 AND 24 can only be built from primes that BOTH have. A circled column is a prime they share; a column with a gap belongs to only one of them.

GCF with Exponents

Same problem, written the short way

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The Steps

- 1 Write each number's primes with **exponents**. The exponent just counts the repeats: $2 \times 2 \times 2 = 2^3$.
- 2 Make a chart with a column for each prime. Put each number's power in its column.
- 3 In each column, pick the **SMALLER** exponent. (A missing prime counts as exponent 0 - so skip it.)
- 4 Multiply the powers you picked. That's the GCF!

Step 1: Write 18 and 24 with exponents

$$18 = 2 \times 3 \times 3 \rightarrow 2^1 \times 3^2$$

$$24 = 2 \times 2 \times 2 \times 3 \rightarrow 2^3 \times 3^1$$

A prime that appears once gets exponent 1. We write it so every number has the same shape.

Steps 2 and 3: Compare exponents, pick the smaller

	prime 2	prime 3
18:	2^1	3^2
24:	2^3	3^1
Pick:	2^1	3^1

(smaller exponent in each column)

Step 4: Multiply what you picked

$$\text{GCF} = 2^1 \times 3^1 = 2 \times 3 = 6$$

TIP Why the SMALLER exponent? In the lined-up picture, 18 has one 2 and 24 has three 2s - so only ONE column of 2s gets circled. The smaller exponent counts the circled columns.

Your Turn: GCF

Find the Greatest Common Factor - use any method you like

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Name: _____

Show your work in each box. Then write the answer on the line. Try a different method for each one if you can!

1 GCF of 12 and 20

Try listing the factors.

GCF = _____

2 GCF of 24 and 36

Try factor trees or exponents.

GCF = _____

3 GCF of 16 and 40

Try the subtraction trick... then the lined-up primes.

GCF = _____

4 GCF of 15 and 28

Careful... what do they share?

GCF = _____

TIP Shortcut check: is one number a multiple of the other? Do they share nothing? Are they close together?

Least Common Multiple (LCM)

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The first number BOTH skip-counts land on

Method 1: List the multiples

- 1 Skip-count by the first number. Write down the multiples.
- 2 Skip-count by the second number. Write those down too.
- 3 Find the FIRST number that shows up in both lists. That's the **LCM**.
- 4 Not there yet? Keep each list going a little longer.

Example: LCM of 6 and 8

6:	6	12	18	24	30	36	...
8:	8	16	24	32	40	48	...

First match: 24

LCM = 24

Check: 24 is in the 6 list (6×4) AND the 8 list (8×3).
The LCM can never be smaller than the bigger number.

Method 2: Factor trees (great for BIG numbers)

Skip-counting by 36 and 48 until they match would take a long time. Factor trees get you there fast - turn to the **next page** for the step-by-step picture.

Word-problem clue words for LCM

'every ___ minutes', 'when will they meet again', 'at the same time', 'the next time both...'. Things are REPEATING and you want the first moment they line up.

TIP **Secret shortcut:** for any two numbers, $GCF \times LCM =$ the two numbers multiplied together.
 $6 \times 8 = 48$, and $GCF 2 \times LCM 24 = 48$. It works every time!

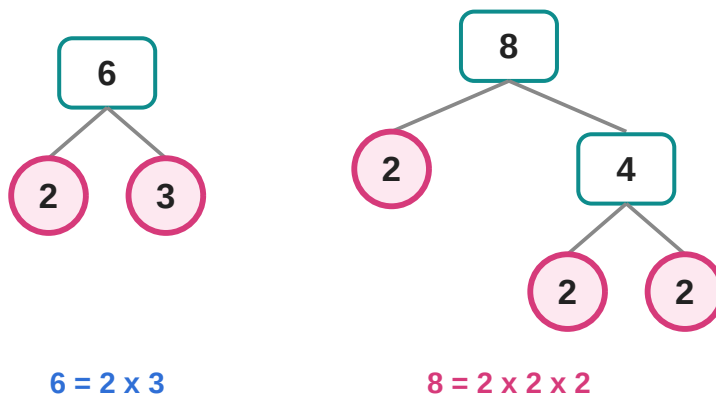
LCM with Factor Trees

Use *EVERY* prime - but count shared ones only once

The Steps

- 1 Make a factor tree for EACH number.
- 2 Write each number's list of primes.
- 3 Write one list above the other and **line up matching primes** in columns - exactly like for the GCF. Circle the shared columns.
- 4 This time use **EVERY** column: circled ones and un-circled ones.
- 5 **Multiply one prime from each column** (a circled column still counts just once). That's the LCM!

Example: LCM of 6 and 8



Line up the primes

$$\begin{array}{rcll} 6 = & 2 & \times & 3 \\ 8 = & 2 & \times & 2 \times 2 \\ \hline \text{LCM} = & 2 & \times & 2 \times 2 \times 3 \end{array}$$

Four columns: 2, 2, 2, 3. LCM = $2 \times 2 \times 2 \times 3 = 24$

$$\text{LCM} = 2 \times 2 \times 2 \times 3 = 24$$

Faster with exponents? The **next page** shows this same problem written with exponents.

TIP Same picture, two answers! Circled columns only = GCF (2). Every column = LCM (24). And $2 \times 24 = 48 = 6 \times 8$ - the shortcut from the last page.

LCM with Exponents

Same problem, written the short way

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The Steps

- 1 Write each number's primes with **exponents**. The exponent just counts the repeats: $2 \times 2 \times 2 = 2^3$.
- 2 Make a chart with a column for each prime. Put each number's power in its column.
- 3 In each column, pick the **BIGGER** exponent. (A missing prime has exponent 0, so the other number's power wins.)
- 4 Multiply the powers you picked. That's the LCM!

Step 1: Write 6 and 8 with exponents

$$6 = 2 \times 3 \rightarrow 2^1 \times 3^1$$

$$8 = 2 \times 2 \times 2 \rightarrow 2^3$$

A prime that appears once gets exponent 1. We write it so every number has the same shape.

Steps 2 and 3: Compare exponents, pick the bigger

	prime 2	prime 3
6:	2^1	3^1
8:	2^3	none (that's 3^0)
Pick:	2^3 (bigger exponent in each column)	3^1

Step 4: Multiply what you picked

$$\text{LCM} = 2^3 \times 3^1 = 8 \times 3 = 24$$

TIP Why the BIGGER exponent? The LCM needs enough 2s to build 8 (three of them) AND enough 3s to build 6 (one). The bigger exponent counts ALL the columns for that prime.

Your Turn: LCM

Find the Least Common Multiple - use any method you like

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Name: _____

Show your work in each box. Then write the answer on the line. Try a different method for each one if you can!

1 LCM of 4 and 10

Try skip-counting.

LCM = _____

2 LCM of 6 and 9

Try lining up the primes.

LCM = _____

3 LCM of 8 and 12

Try exponents.

LCM = _____

4 LCM of 5 and 7

Hmm... do they share anything?

LCM = _____

TIP Shortcut check: if the two numbers share no prime, the LCM is just one times the other. And remember $GCF \times LCM = \text{the two numbers multiplied}$.

Special Cases

1. They share NO prime

8 and 15

GCF = 1 **LCM = A x B**

8 = $2 \times 2 \times 2$ and 15 = 3×5 . Line them up:
not one column matches, so there's nothing to circle. The GCF is just **1**, and the LCM takes every column: $8 \times 15 = 120$.

Both numbers can be composite and still share nothing (9 and 10, 25 and 12).

8 =

2 x 2 x 2

15 =

3 x 5

LCM =

2 x 2 x 2 x 3 x 5

No circles = GCF is 1

2. One divides the other

6 and 18

GCF = the smaller **LCM = the bigger**

18 = 6×3 , so 6 goes into 18 evenly. Line up the primes and **every column of 6 is circled** - the whole of 6 fits inside 18. So GCF = **6**, and the LCM is all of 18's columns: **18**.

Quick check: is the bigger number in the smaller one's skip-count? 6, 12, 18 - yes!

6 =

2 x 3

18 =

2 x 3 x 3

LCM =

2 x 3 x 3

All of 6 circled = GCF is 6

3. They're close together

24 and 28

Gap divides both? Then the gap IS the GCF

28 - 24 = **4**. Does 4 go into 24? Yes. Into 28? Yes. So **GCF = 4**, and LCM = $24 \times 28 / 4 = 168$. No lists, no trees!

Why? Anything that fits both bars must also fit the gap at the end.

blocks of 4

24



28



gap = 4 = one whole block

TIP Always check these three first. Only if none of them fits do you need lists, trees, or exponents.