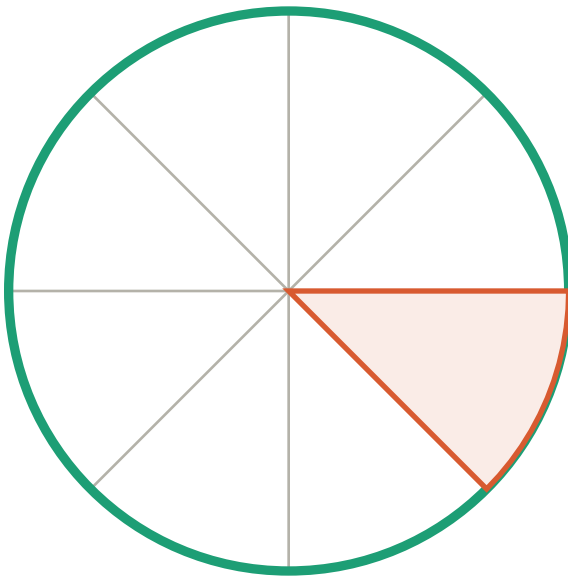


From Around to Inside

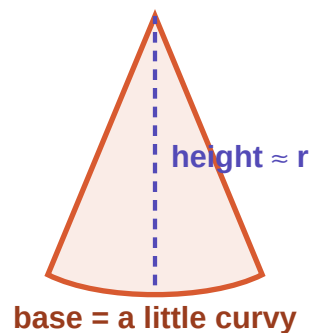
You know $C = 2\pi r$. Now let's find the AREA — the space inside.

Here's the plan: cut the circle into pizza slices. A curvy pizza slice is tricky to measure... but look closely. Each slice is ALMOST a triangle! And triangles are easy:

$$\text{area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$



8 slices — each one almost a triangle



one slice, pulled out

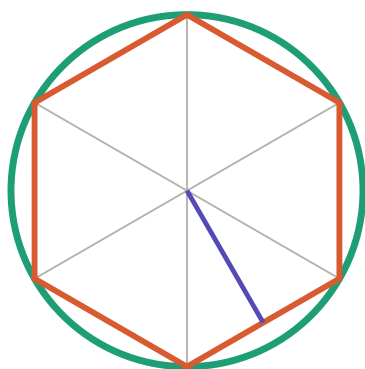
The slice's height reaches from the edge toward the center — almost exactly r .
The slice's base is a little piece of the circumference.

But “almost” isn't good enough for mathematicians. Turn the page!

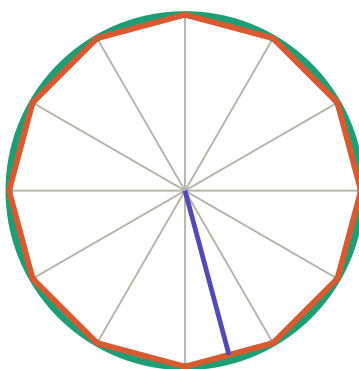
Sneaking Up on the Circle

Straight-sided shapes with more and more sides

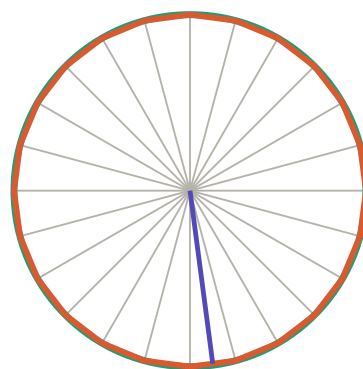
To get rid of “almost,” let's use shapes made **ONLY** of perfect triangles: regular polygons drawn inside the circle. Watch what happens as we give them more and more sides.



6 sides



12 sides



24 sides

— the circle ($C = 2\pi r$) — polygon perimeter — triangle height

Measure the perimeter and the triangle height for each shape ($r = 1$):

Sides	Perimeter	Triangle height
6	6.00	0.87
12	6.21	0.97
24	6.27	0.99
48	6.28	0.998
the circle!	$2\pi \approx 6.283...$	exactly 1 ($= r$)

Do you see the pattern?

The perimeter creeps up to 2π • the triangle height creeps up to r .
With more and more sides, the polygon BECOMES the circle.

The Grand Finale

Adding up all the triangles

A polygon is just triangles glued together, all sharing the center. Add up their areas: every triangle has the same height, and their bases add up to the whole perimeter. So for ANY of our polygons:

$$\text{polygon area} = \frac{1}{2} \times \text{perimeter} \times \text{triangle height}$$

Now use the pattern from page 2. As the sides pile up:

$$\text{perimeter} \rightarrow 2\pi r$$

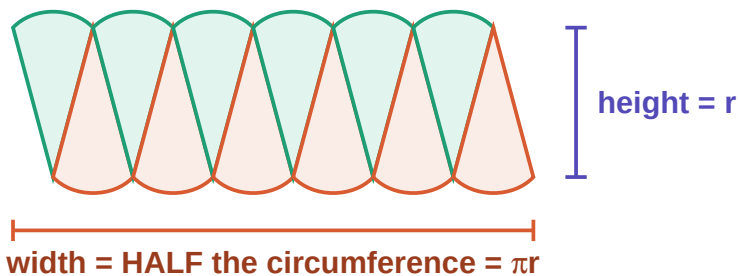
$$\text{height} \rightarrow r$$

$$\text{circle area} = \frac{1}{2} \times 2\pi r \times r$$

$$\mathbf{A = \pi r^2}$$

(the $\frac{1}{2}$ and the 2 cancel each other out — poof!)

Another way to see it: rearrange the slices!



Point half the slices up and half down, and they snuggle into an (almost) rectangle. With skinnier slices the bumps flatten out, and a rectangle's area is:

$$\text{width} \times \text{height} = \pi r \times r = \pi r^2 \quad \checkmark \text{ same answer!}$$

Big idea: straight shapes can sneak up on curvy ones.

Mathematicians call this “the method of exhaustion” — Archimedes used it 2,200 years ago!