

ADVANCED ALGEBRA II

Trigonometry Study Guide

Placement-Test Review & Practice

Goal

Be able to recognize, set up, and solve advanced Algebra II trigonometry problems accurately and efficiently. Unless a problem says otherwise, give exact answers rather than decimal approximations.

Recommended approach

- Work the practice problems without notes first.
- Mark every problem that takes more than about 2-3 minutes, even if you eventually get it right.
- Review the relevant formulas and redo the marked problems later.
- For unit-circle and identity work, aim for exact values without a calculator.

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1. Degrees, Radians, and Angle Measure

Know

- A full circle is 360 degrees = 2π radians.
- Positive angles rotate counterclockwise; negative angles rotate clockwise.
- Coterminal angles differ by 360 degrees or 2π radians.
- For arc length, use $s = r\theta$, with θ measured in radians.

$$\text{Degrees} \rightarrow \text{radians: } \theta^\circ \cdot \pi/180$$

$$\text{Radians} \rightarrow \text{degrees: } \theta \cdot 180/\pi$$

$$s = r\theta$$

Practice

1. Convert 225° to radians.
2. Convert $-7\pi/6$ radians to degrees.
3. Find one positive and one negative angle coterminal with $17\pi/6$.
4. A circle has radius 12 cm. Find the arc length subtended by an angle of $5\pi/6$.

2. Right-Triangle Trigonometry and the Six Trig Functions

Know

- For an acute angle, identify the opposite side, adjacent side, and hypotenuse correctly.
- Use the Pythagorean theorem when a side is missing.
- Know the reciprocal relationships among sine/cosecant, cosine/secant, and tangent/cotangent.

$$\sin \theta = \text{opposite} / \text{hypotenuse}$$

$$\cos \theta = \text{adjacent} / \text{hypotenuse}$$

$$\tan \theta = \text{opposite} / \text{adjacent}$$

$$\csc \theta = 1/\sin \theta \quad \sec \theta = 1/\cos \theta \quad \cot \theta = 1/\tan \theta$$

Practice

1. If $\tan \theta = 5/12$ and θ is acute, find all six trigonometric functions of θ .
2. A right triangle has hypotenuse 17 and one leg 8. Find $\sin \theta$, $\cos \theta$, and $\tan \theta$ for the angle opposite the side of length 8.
3. From a point 40 feet from the base of a building, the angle of elevation to the roof is 58° . Find the building height to the nearest foot.
4. A ladder 18 feet long leans against a wall and makes a 68° angle with the ground. How high up the wall does it reach?

3. The Unit Circle and Exact Trig Values

Know

- Know exact sine and cosine values for multiples of 0° , 30° , 45° , 60° , and 90° .
- The unit-circle point for angle θ is $(\cos \theta, \sin \theta)$.
- Use reference angles and quadrant signs rather than memorizing every point separately.
- Quadrant signs: I all positive; II sine positive; III tangent positive; IV cosine positive.
- Know when $\tan$, $\sec$, $\csc$, or $\cot$ are undefined.

$$(x, y) = (\cos \theta, \sin \theta)$$

$$\tan \theta = \sin \theta / \cos \theta$$

Practice

1. Find $\sin(7\pi/6)$ exactly.
2. Find $\cos(-5\pi/4)$ exactly.
3. Find $\tan(11\pi/6)$ exactly.
4. Find $\sec(3\pi/4)$ exactly.
5. Suppose $\cos \theta = -3/5$ and θ is in Quadrant III. Find $\sin \theta$, $\tan \theta$, and $\csc \theta$.
6. Find all θ in $0 \leq \theta < 2\pi$ for which $\sin \theta = -\sqrt{3}/2$.
7. Find all θ in $0 \leq \theta < 2\pi$ for which $\tan \theta = -1$.

4. Graphs of Trigonometric Functions

Know

- For sine and cosine, identify amplitude, period, horizontal shift, vertical shift, and midline.
- For tangent, there is no amplitude; know the period and vertical asymptotes.
- Recognize the basic shapes of sine, cosine, tangent, secant, cosecant, and cotangent.
- Be able to sketch one period from a transformed equation.

$$y = A \sin(B(x - h)) + D$$

$$y = A \cos(B(x - h)) + D$$

$$\text{Amplitude} = |A| \quad \text{Period}(\text{sine/cosine}) = 2\pi/|B|$$

$$\text{Period}(\text{tangent}) = \pi/|B| \quad \text{Midline: } y = D$$

Practice

1. For $y = 3 \sin(2x) - 1$, identify amplitude, period, horizontal shift, vertical shift, and midline.
2. For $y = -2 \cos(3(x - \pi/6)) + 4$, identify amplitude, period, horizontal shift, and vertical shift.
3. For $y = \tan(x/2) + 2$, identify the period, vertical shift, and the vertical asymptotes surrounding $x = 0$.
4. Sketch one complete period of $y = 2 \sin x + 1$. Label the maximum, minimum, and midline.
5. Write an equation for a cosine function with amplitude 4, period π , midline $y = -2$, and a maximum at $x = 0$.

5. Inverse Trigonometric Functions

Know

- Inverse trig functions return angles.
- $\arcsin x$ has principal range $-\pi/2 \leq y \leq \pi/2$.
- $\arccos x$ has principal range $0 \leq y \leq \pi$.
- $\arctan x$ has principal range $-\pi/2 < y < \pi/2$.
- Do not confuse $\sin^{-1}x$ (inverse sine) with $\csc x$ (reciprocal of sine).
- For compositions such as $\sin(\arccos x)$, draw a right triangle when helpful.

$$y = \arcsin x \text{ means } \sin y = x$$

$$y = \arccos x \text{ means } \cos y = x$$

$$y = \arctan x \text{ means } \tan y = x$$

Practice

1. Evaluate $\arcsin(-\sqrt{3}/2)$ exactly.
2. Evaluate $\arccos(-\sqrt{2}/2)$ exactly.
3. Evaluate $\arctan(-1)$ exactly.
4. Evaluate $\sin(\arccos(5/13))$ exactly.
5. Evaluate $\cos(\arctan(3/4))$ exactly.
6. Evaluate $\tan(\arcsin(-5/13))$ exactly.
7. Evaluate $\arcsin(\sin(5\pi/6))$ exactly.
8. Evaluate $\arccos(\cos(7\pi/4))$ exactly.
9. If $\theta = \arctan(7/24)$, find $\sin \theta$ and $\cos \theta$ exactly.

6. Fundamental Trigonometric Identities

Know

- Know the reciprocal identities.
- Know the quotient identities.
- Know all three Pythagorean identities.
- Know the even/odd identities for sine, cosine, and tangent.
- Recognize cofunction relationships.

$$\csc x = 1/\sin x \quad \sec x = 1/\cos x \quad \cot x = 1/\tan x$$

$$\tan x = \sin x/\cos x \quad \cot x = \cos x/\sin x$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

$$\sin(-x) = -\sin x \quad \cos(-x) = \cos x \quad \tan(-x) = -\tan x$$

$$\sin(\pi/2 - x) = \cos x \quad \cos(\pi/2 - x) = \sin x$$

Practice

1. Simplify $(1 - \cos^2 x)/\sin x$.
2. Simplify $(1 + \tan^2 x)/\sec x$.
3. Simplify $\csc x - \sin x$.
4. If $\sec \theta = -13/5$ and θ is in Quadrant II, find $\sin \theta$, $\tan \theta$, $\csc \theta$, and $\cot \theta$.
5. If $\sin \theta = 8/17$ and θ is in Quadrant II, find $\cos \theta$, $\tan \theta$, $\sec \theta$, and $\cot \theta$.

7. Simplifying and Verifying Trigonometric Identities

Know

- Convert expressions to sine and cosine when that makes the structure clearer.
- Look for Pythagorean identities, common denominators, factoring, and conjugates.
- When verifying an identity, usually manipulate one side until it becomes the other side.
- Domain restrictions matter: an identity is valid wherever both original expressions are defined.

Practice

1. Simplify $(\sec x - \cos x)/\tan x$.
2. Verify: $1/(1 - \sin x) - 1/(1 + \sin x) = 2 \tan x \sec x$.
3. Verify: $\cos x/(1 - \sin x) = \sec x + \tan x$.
4. Simplify $\tan x/\sec x$.
5. Verify: $(1 - \cos x)(1 + \cos x) = \sin^2 x$.
6. Simplify $(\csc x - \sin x)/\cos x$.
7. Verify: $(\sec x + \tan x)(\sec x - \tan x) = 1$.

8. Sum and Difference Identities

Know

- Use sum/difference identities to find exact trig values of non-standard angles such as 15° , 75° , and 105° .
- Pay close attention to the sign in the cosine formulas.
- You may need to find missing sine/cosine values from quadrant information before applying an identity.

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\begin{aligned}\cos(A + B) &= \cos A \cos B - \sin A \sin B \\ \cos(A - B) &= \cos A \cos B + \sin A \sin B \\ \tan(A + B) &= (\tan A + \tan B)/(1 - \tan A \tan B) \\ \tan(A - B) &= (\tan A - \tan B)/(1 + \tan A \tan B)\end{aligned}$$

Practice

1. Find $\sin 75^\circ$ exactly.
2. Find $\cos 15^\circ$ exactly.
3. Find $\tan 105^\circ$ exactly.
4. Find $\cos 105^\circ$ exactly.
5. Suppose $\sin A = 3/5$ and A is in Quadrant II, while $\cos B = 5/13$ and B is in Quadrant IV. Find $\cos(A - B)$.
6. Suppose $\tan A = 1/2$ and $\tan B = 1/3$. Find $\tan(A + B)$, assuming the denominator is nonzero.

9. Double-Angle Identities

Know

- Know the double-angle formulas for sine, cosine, and tangent.
- Know all three useful forms of $\cos(2x)$.
- Choose the form of $\cos(2x)$ that matches the information given.

$$\begin{aligned}\sin(2x) &= 2 \sin x \cos x \\ \cos(2x) &= \cos^2 x - \sin^2 x \\ \cos(2x) &= 2\cos^2 x - 1 \\ \cos(2x) &= 1 - 2\sin^2 x \\ \tan(2x) &= 2\tan x/(1 - \tan^2 x)\end{aligned}$$

Practice

1. Suppose $\sin x = 3/5$ and x is in Quadrant I. Find $\sin 2x$, $\cos 2x$, and $\tan 2x$.
2. Suppose $\cos x = -12/13$ and x is in Quadrant III. Find $\sin 2x$ and $\cos 2x$.
3. Find $\cos(2 \arctan(1/2))$ exactly.
4. Rewrite $1 - 2\sin^2 x$ using a double-angle identity.
5. If $\tan x = 2$, find $\tan 2x$, provided it is defined.
6. Solve $\cos 2x = 1/2$ on $0 \leq x < 2\pi$.

10. Half-Angle Identities

Know

- The square-root form requires choosing a sign based on the quadrant of $x/2$.
- There are several equivalent forms for $\tan(x/2)$.
- Half-angle formulas are especially useful for exact values such as 15° and 22.5° .

$$\begin{aligned}\sin(x/2) &= \pm\sqrt{(1 - \cos x)/2} \\ \cos(x/2) &= \pm\sqrt{(1 + \cos x)/2}\end{aligned}$$

$$\tan(x/2) = \sin x / (1 + \cos x) = (1 - \cos x) / \sin x$$

$$\tan(x/2) = \pm \sqrt{(1 - \cos x) / (1 + \cos x)}$$

Practice

1. Find $\sin 15^\circ$ exactly using a half-angle identity.
2. Find $\cos(3\pi/8)$ exactly.
3. Suppose $\sin \theta = 12/13$ and θ lies in Quadrant II. Find $\tan(\theta/2)$.
4. Find $\sin(\pi/8)$ exactly.
5. Find $\tan(\pi/8)$ exactly.
6. Explain why $\pm$ appears in the square-root half-angle identities but not in the double-angle identities.

11. Solving Trigonometric Equations

Know

- Isolate a trig function, identify the reference angle, determine valid quadrants, and list every solution in the interval.
- Some equations require factoring or using a quadratic substitution.
- Some require an identity before solving.
- If the argument is $2x$, $3x$, or another multiple, solve for that angle first and then solve for x .
- Know the difference between solutions on a finite interval and a general solution using $k \in \mathbb{Z}$.

Practice

1. Solve $2\sin x - 1 = 0$ on $0 \leq x < 2\pi$.
2. Solve $2\cos^2 x - 3\cos x + 1 = 0$ on $0 \leq x < 2\pi$.
3. Solve $2\sin^2 x + \sin x - 1 = 0$ on $0 \leq x < 2\pi$.
4. Solve $\tan(2x) = 1$ on $0 \leq x < 2\pi$.
5. Solve $\cos(2x) = \sin x$ on $0 \leq x < 2\pi$.
6. Give the complete general solution for $\sin x = -\sqrt{2}/2$.
7. Solve $2\cos x + \sec x = 3$ on $0 \leq x < 2\pi$.
8. Solve $\sin^2 x = 3/4$ on $0 \leq x < 2\pi$.

12. Law of Sines

Know

- Use the Law of Sines when you know an angle and its opposite side.
- It is especially useful for ASA, AAS, and SSA triangles.
- SSA is the ambiguous case: there may be zero, one, or two possible triangles.
- When using inverse sine in an SSA problem, check whether the supplementary angle also creates a valid triangle.

$$a/\sin A = b/\sin B = c/\sin C$$

Practice

1. In triangle ABC, $A = 42^\circ$, $B = 68^\circ$, and $a = 12$. Find C , b , and c .
2. In triangle ABC, $A = 30^\circ$, $a = 7$, and $b = 10$. Determine all possible triangles.
3. In triangle ABC, $A = 25^\circ$, $a = 8$, and $b = 20$. Determine whether zero, one, or two triangles exist.
4. In triangle ABC, $B = 57^\circ$, $C = 71^\circ$, and $c = 15$. Find A , a , and b .
5. Explain why the Law of Sines generally cannot immediately solve a triangle when you are given only three side lengths.

13. Law of Cosines

Know

- Use the Law of Cosines for SAS or SSS information.
- When solving for a side, the known angle must be the included angle between the other two sides.
- When solving for an angle from three sides, isolate the cosine before applying arccos.
- The Pythagorean theorem is the special case when the included angle is 90° .

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

Practice

1. Suppose $a = 8$, $b = 11$, and $C = 53^\circ$. Find c .
2. A triangle has side lengths 7, 9, and 12. Find the angle opposite the side of length 12.
3. Suppose $b = 6$, $c = 10$, and $A = 120^\circ$. Find a exactly.
4. A triangle has sides 5, 7, and 9. Find its largest angle.
5. State whether you would begin with the Law of Sines or Law of Cosines for each: (a) $A=50^\circ$, $B=65^\circ$, $a=8$; (b) $a=8$, $b=10$, $C=40^\circ$; (c) $a=7$, $b=9$, $c=12$; (d) $A=35^\circ$, $a=10$, $b=13$.

14. Area of a Non-Right Triangle

Know

- If two sides and their included angle are known, use one-half times the product of the sides times the sine of the included angle.
- The angle used must be the angle between the two sides in the formula.

$$K = \frac{1}{2} ab \sin C = \frac{1}{2} bc \sin A = \frac{1}{2} ac \sin B$$

Practice

1. Find the area of a triangle having sides 10 and 14 with included angle 37° .
2. Find the exact area of a triangle with $b = 12$, $c = 9$, and $A = 60^\circ$.
3. A triangle has sides 7 and 11 with included angle 125° . Find its area to the nearest tenth.
4. Why must the angle in $K = \frac{1}{2} ab \sin C$ be the angle between sides a and b ?

15. Mixed Trigonometry Challenge

These problems deliberately do not tell you which technique to use.

1. Given $\cos \theta = -4/5$ and $\pi < \theta < 3\pi/2$, find $\sin 2\theta$ exactly.
2. Find $\sin 105^\circ$ exactly.
3. Solve $2\cos^2 x + \cos x - 1 = 0$ on $0 \leq x < 2\pi$.
4. Simplify $(1 - \sin^2 x)/\cos x$.
5. Evaluate $\arcsin(\sin(7\pi/6))$ exactly.
6. A triangle has sides 8 and 13 with an included angle of 47° . Find the third side.
7. If $\tan \theta = 3/4$ and θ is in Quadrant III, find $\cos 2\theta$.
8. Solve $\sin 2x = \sin x$ on $0 \leq x < 2\pi$.
9. Find $\tan 15^\circ$ exactly.
10. If $\sin \theta = 5/13$ and θ is in Quadrant II, find $\cos(2\theta)$ exactly.
11. Prove the identity $(1 - \cos 2x)/2 = \sin^2 x$.
12. A triangle has $A = 41^\circ$, $a = 12$, and $b = 17$. Determine all possible values of B and state how many triangles exist.
13. Find all x in $0 \leq x < 2\pi$ satisfying $2\sin x \cos x = \sqrt{3}/2$.
14. Evaluate $\cos(\arcsin(8/17))$ exactly.
15. Write an equation for a sine function with amplitude 3, period $\pi/2$, midline $y = 4$, and no horizontal shift.

Mastery Checklist

Before the placement exam, you should be able to say yes to each statement without relying heavily on notes.

- ☐ I can convert between degrees and radians quickly.
- ☐ I can find coterminal and reference angles.
- ☐ I know the unit circle and exact trig values without looking them up.
- ☐ I know all six trig functions and their reciprocal relationships.
- ☐ I can find trig values using quadrant information.
- ☐ I can graph transformed sine, cosine, and tangent functions.
- ☐ I know the principal ranges of inverse sine, inverse cosine, and inverse tangent.
- ☐ I can evaluate compositions involving inverse trig functions.
- ☐ I know the reciprocal, quotient, and Pythagorean identities.
- ☐ I can use even/odd and cofunction identities.
- ☐ I can simplify and verify multi-step trig identities.
- ☐ I know the sum and difference identities.
- ☐ I know the double-angle identities.
- ☐ I know the half-angle identities and can determine the correct sign.
- ☐ I can solve trig equations over a specified interval.
- ☐ I can give general solutions to trig equations.
- ☐ I know when to use Law of Sines versus Law of Cosines.
- ☐ I can recognize and correctly handle the SSA ambiguous case.
- ☐ I can find the area of a non-right triangle using trigonometry.
- ☐ I can solve mixed trig problems without being told which technique to use.

Suggested priority for a difficult placement test

- Unit circle and exact values
- Inverse trig principal values
- Identities and identity verification
- Sum/difference, double-angle, and half-angle formulas
- Trig equations
- Law of Sines ambiguous case
- Law of Cosines
- Mixed problems where the method is not identified