

ADVANCED ALGEBRA II

Sequences and Series Study Guide

Placement Test Review - Student Practice Packet

Goal: Recognize sequence patterns, write explicit and recursive rules, compute finite and infinite sums, use sigma notation, and solve multi-step placement-test problems without being told which formula to use.

1. Sequence Vocabulary and Notation

- A sequence is an ordered list of numbers. Each number is a term.
- a_n means the n th term; a_1 is the first term.
- A finite sequence ends. An infinite sequence continues indefinitely.
- An explicit formula gives a_n directly from n . A recursive formula defines terms using earlier terms.

$$a_1, a_2, a_3, \dots, a_n$$

Practice

1. For the sequence 7, 12, 17, 22, ..., state a_1 , a_4 , and a_{10} if the pattern continues.
2. List the first five terms of $a_n = 3n^2 - 2n$.
3. List the first six terms of the recursive sequence $a_1 = 4$ and $a_n = a_{(n-1)} - 3$ for $n \geq 2$.
4. A sequence has $a_3 = 11$ and $a_7 = 27$. Explain what those two statements mean in words; do not assume yet that the sequence is arithmetic.

2. Arithmetic Sequences

- An arithmetic sequence has a constant difference d between consecutive terms.
- $$a_n = a_1 + (n - 1)d$$
- To test whether a sequence is arithmetic, subtract consecutive terms.
 - If you know two nonconsecutive terms, use their positions as well as their values to find d .

Practice

1. Find the common difference and an explicit formula for 18, 13, 8, 3, ...
2. Find a_{40} for the arithmetic sequence with $a_1 = -7$ and $d = 6$.
3. An arithmetic sequence has $a_5 = 17$ and $a_{13} = 49$. Find d , a_1 , and a_{30} .
4. Which term of the sequence 11, 18, 25, 32, ... is 277?
5. Insert four arithmetic means between 6 and 36.

3. Arithmetic Series and Partial Sums

- A series is the sum of the terms of a sequence.
- S_n means the sum of the first n terms.

$$S_n = n/2 (a_1 + a_n)$$

$$S_n = n/2 [2a_1 + (n - 1)d]$$

- Use whichever form matches the information you have.

Practice

1. Find the sum of the first 35 terms of 4, 9, 14, 19, ...
2. Find $21 + 25 + 29 + \dots + 137$.
3. An arithmetic series has $a_1 = 12$, $d = -3$, and $S_n = -330$. Find n .
4. The 8th term of an arithmetic sequence is 29 and the 20th term is 77. Find the sum of the first 30 terms.
5. Find the sum of all positive multiples of 7 less than 500.

4. Geometric Sequences

- A geometric sequence has a constant ratio r between consecutive terms.

$$a_n = a_1 r^{(n - 1)}$$

- To test whether a sequence is geometric, divide consecutive nonzero terms.
- A negative ratio causes signs to alternate.

Practice

1. Find the common ratio and an explicit formula for 160, 80, 40, 20, ...
2. Find a_9 if $a_1 = 6$ and $r = -2$.
3. A geometric sequence has $a_3 = 20$ and $a_6 = 160$. Find all real possibilities for r and determine the corresponding a_1 values.
4. Which term of 3, 12, 48, 192, ... equals 196,608?
5. A geometric sequence has $a_2 = 18$ and $a_5 = 486$. Find an explicit formula.

5. Finite Geometric Series

$$S_n = a_1(1 - r^n)/(1 - r), \quad r \neq 1$$

- A finite geometric series can be summed even when $|r| \geq 1$.
- Be careful to identify n correctly, especially when the first exponent is not 0.

Practice

1. Find the sum of the first 8 terms of $5 + 15 + 45 + 135 + \dots$.
2. Find $256 - 128 + 64 - 32 + \dots$ through the 9th term.
3. Find the sum $3 + 6 + 12 + \dots + 3072$.
4. A geometric series has $a_1 = 7$, $r = 3$, and $S_n = 68887$. Find n .

5. Without adding term by term, evaluate $2^0 + 2^1 + 2^2 + \dots + 2^{12}$.

6. Infinite Geometric Series and Convergence

- An infinite geometric series converges only when $|r| < 1$.

$$S_{\infty} = a_1/(1 - r), \quad |r| < 1$$
- If $|r| \geq 1$, the infinite series diverges; it does not have a finite sum.
- A repeating decimal can often be written as an infinite geometric series.

Practice

- Determine whether $12 + 6 + 3 + 3/2 + \dots$ converges. If it does, find its sum.
- Determine whether $5 - 10 + 20 - 40 + \dots$ converges. Justify your answer.
- Find the exact sum of $9 + 3 + 1 + 1/3 + \dots$.
- Write $0.272727\dots$ as an infinite geometric series, then express it as a fraction.
- Find x if $4 + 4x + 4x^2 + 4x^3 + \dots = 10$, assuming the series converges.

7. Sigma Notation

- Sigma notation is a compact way to write a sum.

$$\text{sum from } k=m \text{ to } n \text{ of } f(k) = f(m) + f(m+1) + \dots + f(n)$$
- The index variable is a placeholder; k , i , j , or another letter may be used.
- Always check the lower and upper bounds carefully.

Practice

- Expand and evaluate: sum from $k=1$ to 6 of $(3k - 2)$.
- Expand the first four terms of: sum from $n=2$ to 8 of $5(1/2)^{(n-2)}$. Do not evaluate the entire sum.
- Write $7 + 11 + 15 + \dots + 99$ using sigma notation.
- Write $3 + 12 + 48 + 192 + \dots + 3(4^9)$ using sigma notation.
- Evaluate: sum from $k=1$ to 20 of $(2k + 1)$ without adding all 20 terms individually.

8. Recursive and Explicit Formulas

- An explicit formula lets you jump directly to any term.
- A recursive formula needs a starting value and a rule relating one term to earlier terms.
- For arithmetic sequences: $a_n = a_{(n-1)} + d$.
- For geometric sequences: $a_n = r a_{(n-1)}$.

Practice

- Write both an explicit and recursive formula for $14, 19, 24, 29, \dots$.
- Write both an explicit and recursive formula for $81, -27, 9, -3, \dots$.
- Convert $a_1 = 11, a_n = a_{(n-1)} + 7$ into an explicit formula.
- Convert $a_1 = 5, a_n = -2a_{(n-1)}$ into an explicit formula.

5. The recursive sequence is $a_1 = 2$, $a_2 = 5$, and $a_n = a_{(n-1)} + a_{(n-2)}$. Find a_3 through a_8 . Is it arithmetic or geometric?

9. Recognizing Patterns and Building Formulas

- Do not automatically assume every sequence is arithmetic or geometric.
- For polynomial sequences, finite differences can reveal the degree: constant first differences suggest linear; constant second differences suggest quadratic.
- Sometimes the pattern depends on parity, alternating signs, powers, or a recursive rule.

Practice

1. Classify each as arithmetic, geometric, or neither: (a) 2, 6, 18, 54, ...; (b) 4, 9, 14, 19, ...; (c) 1, 4, 9, 16, ...
2. For 3, 8, 15, 24, 35, ... compute first and second differences. Guess an explicit formula and verify it for the first five terms.
3. Find an explicit formula for 1, -4, 9, -16, 25, ...
4. Find an explicit formula for 2, 5, 10, 17, 26, ...
5. A sequence begins 1, 1, 2, 3, 5, 8, 13, ... Give a recursive definition and explain why there is no constant difference or ratio.

10. Applications: Growth, Decay, Savings, and Repeated Change

- Repeated equal additions usually lead to arithmetic sequences.
- Repeated equal percentage changes usually lead to geometric sequences.
- Always identify what a_1 represents and whether the first change happens before or after the first term.

Practice

1. A theater has 24 seats in the first row and 3 more seats in each successive row. There are 18 rows. How many seats are in the theater?
2. A ball rebounds to 72% of the height from which it falls. It is dropped from 20 feet. Write a geometric sequence for the maximum rebound heights and find the height of the 6th rebound.
3. A bacteria culture starts with 800 cells and doubles every 3 hours. Write a sequence for the population at 3-hour intervals and find the population after 24 hours.
4. A student deposits \$50 in month 1, \$60 in month 2, \$70 in month 3, and continues the pattern for 18 months. How much is deposited in total?
5. A machine worth \$48,000 loses 15% of its value each year. Write an explicit sequence for its value after n years and determine after how many whole years its value first falls below \$20,000.

11. Placement-Test Mixed Challenge

These problems deliberately do not tell you whether to use an arithmetic formula, geometric formula, recursion, sigma notation, or another strategy.

1. An arithmetic sequence satisfies $a_7 = 31$ and $a_{19} = 79$. Find S_{40} .
2. A geometric sequence satisfies $a_4 = 54$ and $a_7 = 1458$. Find a_1 and a_{10} .

3. Find the exact value of $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$.
4. Find n if $6 + 10 + 14 + \dots + a_n = 966$.
5. Write the sum $5 + 15 + 45 + \dots + 5(3^{11})$ in sigma notation, then find its exact value.
6. An infinite geometric series has sum 24 and first term 18. Find its common ratio.
7. The 3rd term of an arithmetic sequence is 14 and the sum of its first 12 terms is 330. Find a_1 and d .
8. For the sequence 2, 6, 12, 20, 30, 42, ... find a formula for a_n .
9. A geometric sequence has positive terms. Its 2nd term is 12 and its 5th term is 324. Find the sum of its first 7 terms.
10. A runner completes 10 miles in week 1 and increases weekly mileage by 8% each week. Is total mileage over the first 12 weeks modeled by a sequence or a series? Write the appropriate expression and find the total mileage to the nearest tenth.

12. Mastery Checklist

- ☐ I can distinguish a sequence from a series.
- ☐ I can use and interpret a_n , a_1 , d , r , S_n , and sigma notation.
- ☐ I can recognize arithmetic and geometric sequences quickly.
- ☐ I can write explicit and recursive formulas for arithmetic sequences.
- ☐ I can write explicit and recursive formulas for geometric sequences.
- ☐ I can find missing information from two nonconsecutive terms.
- ☐ I can find the n th term and solve for which term has a given value.
- ☐ I can compute arithmetic partial sums.
- ☐ I can compute finite geometric sums.
- ☐ I know exactly when an infinite geometric series converges.
- ☐ I can find the sum of a convergent infinite geometric series.
- ☐ I can convert repeating decimals into geometric series and fractions.
- ☐ I can expand, interpret, and write sigma notation.
- ☐ I can identify patterns that are neither arithmetic nor geometric.
- ☐ I can use finite differences to recognize simple quadratic sequences.
- ☐ I can translate growth, decay, saving, and repeated-change situations into sequences or series.
- ☐ I can solve mixed problems without being told which formula applies.

Placement-test reminders: Check whether n starts at 0 or 1; distinguish a_n from S_n ; do not use the infinite-sum formula unless $|r| < 1$; and when given two nonconsecutive terms, account for the number of steps between them.